International Journal of Research and Technology in Electrical Industry

journal homepage: ijrtei.sbu.ac.ir

Shahid Beheshti University

Development a New Flexibility Index Suitable for Power System Operational Planning

Homayoun Berahmandpour [1,*], Shahram Montaser Kouhsari[2], Hassan Rastegar[2]

[1] *Head of Bulk Power Transmission Technology Center, Niroo Research Institute, Tehran, Iran*
[2] *Electrical Engineering Department, Amirkabir university, Tehran, Iran*

ARTICLEINFO

Article history:
Received 28 December 2022
Received in revised form 19 February 2023
Accepted 24 February 2023

Keywords:
Power system flexibility
Flexibility index
Operational planning
Renewable resource integration

ABSTRACT

Power system flexibility is an important characteristic in both power system planning and operation, which should be evaluated and maintained in the desired value. On the other hand, more renewable energy integration leads to increasing uncertainty and variability in the power system. Therefore, the power system should have the sufficient ability to overcome the adverse effects of uncertainty and variability named as flexibility, which should be improved with suitable tools such as adequate reserve, fast ramp-up/down generation sources and suitable energy storage capacity. Power system flexibility evaluation is the main task that needs suitable indices to indicate the level of system flexibility correctly. In the current paper, a well-known system flexibility index named normalized flexibility index, which is used for power system planning horizon is modified to use for the operational planning time zone. In this concept, the flexibility index is separated into two components, each of them indicating the ability of the power system to withstand upward/downward net-load uncertainty and variability. In the further, this is shown these two components are the same as the upward/downward system reserve and can be converted to economic value simply. So, this concept facilitates the economic trade-off between operation cost and system flexibility, improving cost to achieve the best level of system flexibility.

1. Introduction

As the world shift to more renewable energy, especially variable ones such as wind and solar, a paradigm shift in the power sector has gradually taken place to meet the transition. In particular, is the growing trend of more focus on the so-called "Power System Flexibility" in the academic and industrial sectors in this field.

Power system flexibility is an important criterion that shows the capability of the power system to withstand uncertainty and variability arising from different resources such as electrical load behaviour, Variable Renewable Energy (VRE) output power, power market player actions and so on. One of the comprehensive definitions of this important power system specification can be found in [1].

The term flexibility describes the ability of a power system to cope with variability and uncertainty in both generation and demand, while maintaining a satisfactory level of reliability at a reasonable cost, over different time horizons.

Four main concepts of flexibility are underlined in this definition. The main concept is “uncertainty and variability,” which establishes the main framework for

---

* Corresponding author
E-mail address: hberahmandpour@aut.ac.ir
https://www.orcid.org/0000-0001-6398-7234
http://dx.doi.org/10.52547/ijrtei.1.1.29

power system flexibility studies and should be modeled for each source of uncertainty correctly (such as load or renewable resource output). Also, uncertainty and variability are extended to both the generation and demand sides. Where the important key item “reliability at a reasonable cost” shows the need for the economic trade-off between reliability and operation costs to find the best level of flexibility in each power system and each operation situation (as generation point or network configuration). Finally, the last item shows the wide time range for flexibility study from real-time operation up to long term extension planning time horizons. Fig. 1 shows the main features of the flexibility study in the different time horizons.

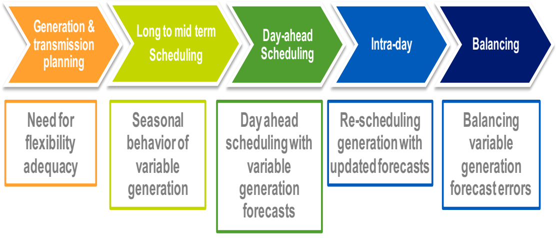

Fig. 1. Flexibility challenges in different time zones

According to the International Energy Agency, the flexibility of a power system refers to "the extent to which a power system can modify electricity production or consumption in response to variability, expected or otherwise" [2]. Another source described it as "the modification of generation injection and/or consumption patterns in reaction to an external signal (price signal or activation) in order to provide a service within the energy system" [3]. But the first definition seems to be more comprehensive and perfect.

The main challenge with insufficient power system flexibility in the operation time horizon is generation/load unbalance defect. It can appear as an unpermitted frequency deviation, unwanted load shedding and renewable curtailment, all of which expose extra costs to the power system planning and operation. On the other hand, increasing flexibility puts extra costs on the power system. So, balancing these two main costs leads to the economic level of power system flexibility. Fig. 2 shows the main tools for power system flexibility improvement concerning their costs. In this way, power system flexibility evaluation is one of the main tasks in the power system flexibility study. Where the power system flexibility level can be determined by a suitable and meaningful measure named as flexibility index.

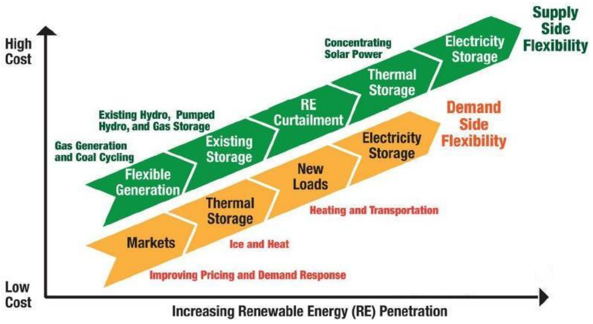

Fig. 2 Differenet tools for flexibility improvement

The current paper is complementary to the concept described in [4] to introduce the new concept as up/down components of the flexibility index. In this way and in the second part, a short review of the main approach for the flexibility evaluation is described. Part 3 presents the concept of operational planning flexibility which is the main approach of the current paper. One of the well-known flexibility indices for power system planning time horizon as “Normalized Flexibility Index (NFI)” is determined and discussed in part 4. The main contribution of this paper is to modify this index to use for operational planning time horizon, where it leads to a new flexibility index for power system flexibility evaluation and as said before, to separate the proposed index to up/down components each of them to evaluate the up/down system flexibility. The physical concept for the new index is explained in part 6. Where this is an important contribution that leads to the economic value of the flexibility index. Simulation and result analysis are presented in part 7 and finally, part 8 includes the conclusion.

2. Flexibility evaluation

Increasing the penetration of variable renewable generation in power systems worldwide is one of the main reasons for uncertainty and variability rapid growth in power systems and therefore to more pay attention to power system flexibility. It forces sufficient flexibility to overcome the uncertainty and variability that arise from these types of generation. Traditional capacity adequacy planning techniques have been supplemented with integration studies, which have been carried out in power systems with high targets for renewable generation. These have highlighted the increased variability that a system may experience in the future. As the system generation planning techniques evolve with the challenge of integrating variable generation, the flexibility of a system to manage periods of high variability needs to be assessed [5].

The first requirement for power system flexibility evaluation is to develop a suitable measure/index to quantify system flexibility level. The flexibility index should determine the ability of the power system to overcome both uncertainty and variability specifications in both generation and demand sides. The level of flexibility of two systems or two operating points of one system can be compared with this index and the improvement of the flexibility level by corrective and improvement actions can be obtained by increasing this index in the next step.

Generally, the accessible generation capacity and ramp rate capability of the system generation are two main characteristics of flexibility on the generation side. Therefore, generation units providing dynamic reserve by fast ramp rate characteristics are the essential tools to provide flexibility on the generation side. On the other hand, the energy stored in energy storage systems or accessible energy due to limited energy sources such as hydroelectric power plants or pumped-storage systems can help to improve system flexibility.

To have a clear overview of power system flexibility and also to compare the different power systems flexibility levels or one power system flexibility level in different situations, it needs to evaluate power system flexibility by suitable flexibility index. This index should indicate the level of power system flexibility properly and also it should be converted to an economic value to facilitate the cost/benefit analysis of the power system flexibility in different levels. The insufficient ramping resource expectation (IRRE) metric is a well-known index proposed to measure power system flexibility mainly for use in long-term planning and is derived from traditional generation adequacy metrics [5]. Compared to existing generation adequacy metrics, flexibility assessment is more data-intensive. A flexibility metric can identify the time intervals over which a system is most likely to face a shortage of flexible resources, and can measure the relative impact of changing operational policies and the addition of flexible resources. The flexibility of the desired system with increasing penetrations of variable generation is assessed and the results highlight the time horizons of increased and decreased risk associated with the integration of the variable generation systems [6].

A large number of metrics and indicators are currently used in power systems to measure the power system different abilities such as reliability, security, transient stability and so on. Power system flexibility indices are also the measures for the ability of the power system against uncertainty and variability. The indicators such as Loss Of Load Expectation (LOLE) and Expected Energy Not Served (EENS) are used for power system flexibility evaluation in a planning context to determine the adequacy of future systems [7]. These metrics are very similar to the power system reliability indices and are categorized as probabilistic indices. On the other hand, some of the indices categorized as deterministic are calculated by the system parameters in each situation of the power system. Four main parameters of system generation are used to form these types of indices as system storage energy (ε), system power capacity (π), system ramp rate capability (ρ) and ramp duration time (δ) [6]. In a dominant approach for deterministic indices, the area bounded by the permitted values of these parameters forms the locus of system generation points which can be found to respond to net load uncertainty and variability. Fig. 3 shows the concept of the flexibility index based on the permitted area for system generation points bounded by the four mentioned parameters [7].

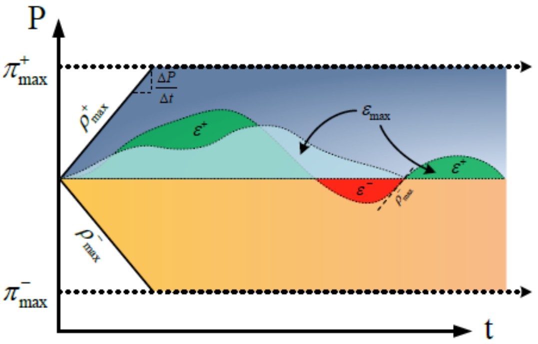

Fig.3. Permitted area concept for flexibility index

Fig. 4 shows an approach on the probabilistic index as Lack Of Ramp Probability (LORP) [8].

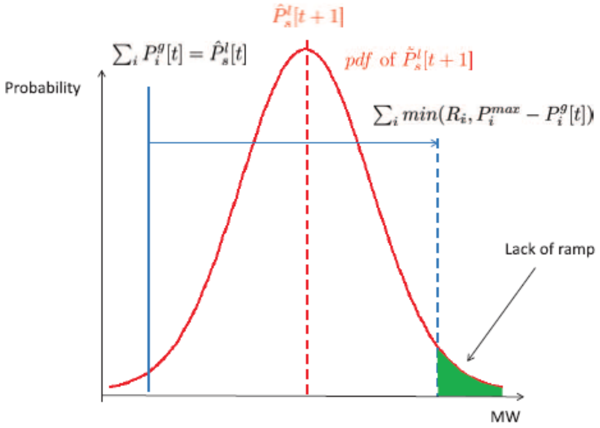

Fig. 4. LORP flexibility index

Here the probabilistic behaviour of the load power at the time (t+1), is shown by the normal probability distribution function. LORP shows the probability of the load power at the time (t+1) cannot be covered by the generation system capability because of the defect in the generation ramp rate. The green area in Fig. 4 shows this probability and correspondingly this index.

System Capability Ramp (SCR) [9], is another probabilistic index. This index is introduced as:

$$\mathrm{SCR}_{t+\Delta t} = \sum_{i=1}^{n} (A_{i,t}\, O_{i,t}) \min\, (P^{max}_{i,t} - P_{i,t}, Ramp_i \Delta t) \quad (1)$$

Where A and O are the availability and situation of the unit i in time t respectively. P and Ramp are also the generation and ramp rate of this unit in time t. The main probabilistic nature of this approach is the unit availability which should be calculated by the conventional methods such as Markov chain-based capacity state model [9].

Ramping capability Shortage Expectation (RSE) [10] is another probabilistic index. In this approach, the risk of the ramping capability shortage is quantified where RSE is defined as the sum of the probabilities that the ramping capability requirement will be not satisfied. Clearly by decreasing RSE system generation flexibility will be increased. (Unlike SCR)

Finally, Ramping capability Shortage Probability (RSP) [11] is another well-known flexibility index used to evaluate system flexibility level. The RSP at time t is defined as the sum of the probabilities that net load variation during an interval between t−Δt and t could not be covered by the ramping capability of the system.

Now by the short review about the different approaches on the generation system flexibility indices, the operational planning flexibility concept which is the main concept of the current paper, is described.

3. Operational planning flexibility

Operational flexibility is an important property of electric power systems and is essential for mitigating generation/load unbalance comes from uncertainty and variability in short/mid-term power system operation. The availability of sufficient operational flexibility is a necessary prerequisite for effective power system operation, especially with grid integration of large shares of fluctuating power resources mainly variable renewable energy sources [12].

Operational planning includes the time horizon in about a few minutes to one day or one week ahead (Fig. 5).

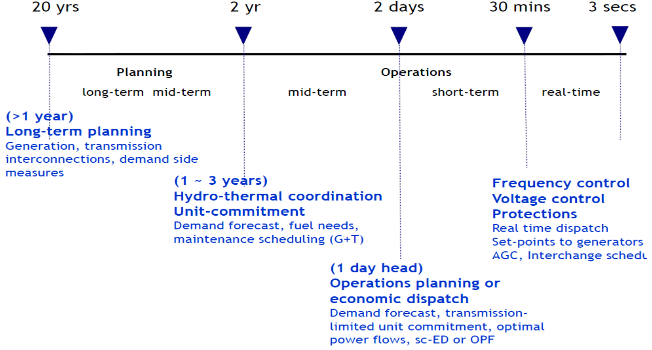

Fig. 5. Operational planning time zone

Economic Dispatch (ED) and Unit Commitment (UC) are two main tasks in this time horizon. In a shorter time scale (maybe a few minutes to 1 hour), economic dispatch is the main task for power system operational planning where the time horizon is less than generation unit start-up and shut-down times. The main focus of the current paper is in this time horizon to introduce the suitable index for power system flexibility evaluation.

Dependency to operation point is the main requirement for power system flexibility indices in the operational flexibility evaluation with respect to the planning evaluation horizon. This approach can be seen in the LORP index in Fig.4. Where the ability of the generation system to respond to net load uncertainty directly depends on the current system generation point.

In the next part the well-known flexibility index, which is mainly used for power system planning horizon is described and then this approach is modified for operational planning horizon by considering the current system generation point as the main requirement of the flexibility index in this time horizon.

4. Normalized Flexibility Index (NFI)

NFI is a well-known flexibility index mainly used for the evaluation of power system flexibility in planning time horizons [13]. This index is defined for one generation unit as:

$$flex(i) = \frac{0.5\left(P_{max}(i) - P_{min}(\mathrm{i})\right) + 0.5Ramp(i)\Delta t}{P_{max}(i)} \qquad (2)$$

P_{min}, P_{max} are the minimum and maximum bounds of the unit generation and Ramp in the mean of up/down generation ramp rate. Δt is the desired time interval for system generation response to the uncertainty and variability caused by the different factors. The high value of the unit flexibility index shows more capability to withstand uncertainty and variability in both the generation and demand sides. In the current paper, (2) is rewritten as (3) to define two items of the flexibility index.

$$flex(i) = \frac{0.5\left(P_{max}(i) - P_{min}(\mathrm{i})\right)}{P_{max}(i)} + \frac{0.5Ramp(i)\Delta t}{P_{max}(i)} \qquad (3)$$

The first item shows the capacity of the unit generation, the main capability to overcome generation/load uncertainty. This is the same parameter (π) mentioned in part 2. On the other hand, the second item shows the capability of the generation unit to withstand generation/load variability. Here the two parameters as (ρ) and (δ) are considered to form the second item. So, three parameters (π), (ρ) and (δ) are used to form this index.

The generation system flexibility index can be determined by combining all the generation unit indices. In this way all of these indices are combined by their generation capacities as the weighting factors [13]:

$$FLEX = \frac{\sum_{i=1}^{n} flex(i)P_{max}(i)}{\sum_{i=1}^{n} P_{max}(i)} \qquad (4)$$

So, the generation units with the higher generation capacity have more effect on the system flexibility index. As said before, this index is used for flexibility evaluation in the planning time horizon. But there are serious criticisms of using this approach for operation flexibility evaluation, mainly because of no appearance of the current operation point of the generation units in (2). The main challenges to using (2) and (4) for operational planning purposes can be summarized as below:

1-As can be seen, the flexibility index is independent of the current generation unit/generation system operation point. So, the current generation point has no effect on the flexibility index.

2-The ramp rate capability may be limited because of the up/down unit generation constraints. When the current generation point (P_g) is near the maximum or minimum boundaries, it may limit the ramp rate capability. As is shown in Fig. 6, if Rampup Δt>(P_{max}-P_g) or Rampdn Δt>(P_g-P_{min}), P_{max}-P_g or P_g-P_{min} should be considered as the boundaries of the generation ramp capability (dotted red/green lines respectively).

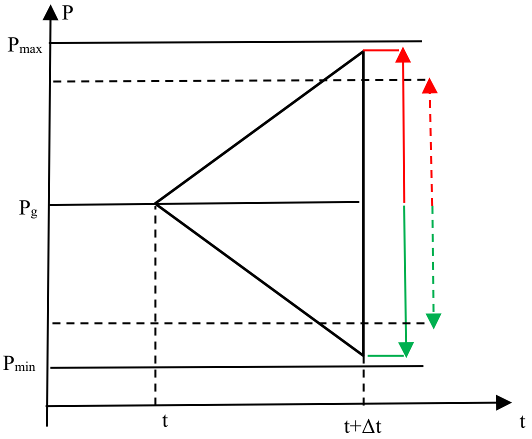

Fig. 6. Proposed flexibility index concept

3- The weighting factors of the two items in (3) are equal which has no acceptable reason.

Now in the next part, the contribution of this paper to extend NFI definition for use in power system operational planning is described. The main modification is to involve the current generation point to show the effect of the unit generation situation in the flexibility index.

5. NFI improvement for operational planning

As said before, the main approach in NFI modification is to consider the generation operation point in the flexibility index. This concept yields to combining the two items in the flexibility formulation described by (3). By considering $P_g(i)$ as the operation point of the generator (i) at the time (t), the generation point is bounded by $P_g(i)$+RampupΔt and $P_g(i)$-RampdnΔt in the time (t+Δt) which are shown by solid red/green lines in Fig. 6. But if $P_g(i)$+RampupΔt is more than $P_{max}(i)$, the upbound is limited to $P_{max}(i)$. Also, if $P_g(i)$-RampupΔt is less than $P_{min}(i)$, the downbound is limited to $P_{min}(i)$. In this case, the up/down bounds of the generator operation points are shown as the dotted red/green lines in Fig. 6.

By this approach, the modification of (3) to deal with the current generation point is shown by (5).

$$flex(i) = \min\left(Rampup(i)\Delta t, \left(P_{max}(i) - P_g(i)\right)\right. + \min\left(Rampdn(i)\Delta t, (P_g(i) - P_{min}(i))\right) \tag{5}$$

Here the two up/down bounds of the generator operation point in time (t+Δt)) are considered as the first and the second items. So, the two characteristics as system generation capacity (π) and system ramp up/down (ρ) are combined together, eliminating the weighting factors in (3). Also, the up capacity/ramp and down capacity/ramp are separated to show the ability of the generation unit in each of the two directions. As can be seen, dividing by the generation capacity ($P_{max}(i)$) is ignored in the new formulation. The reason will be illustrated later. Now the two items of (5) can be separated and called up/down flexibility components of the flexibility index.

$$Up\_Comp(i) = \min\left(Rampup(i)\Delta t, \left(P_{max}(i) - P_g(i)\right)\right. \tag{6}$$

$$Dn\_Comp(i) = \min\left(Rampdn(i)\Delta t, (P_g(i) - P_{min}(i))\right) \tag{7}$$

The up/down components are very meaningful and useful concepts that show the ability of the generation unit to cope with the net load uncertainty and variability in up and down directions respectively.

In the next step, the combination of the generator flexibility indices is described to calculate the system generation flexibility index. As can be seen in (3) and (4), $P_{max}(i)$ is in the denominator. On the other hand, in (4) flex(i) is multiplied by $P_{max}(i)$ again. Clearly, we can eliminate $P_{max}(i)$ both in (3) and (4). So, no need to divide (5), (6) and (7) by $P_{max}(i)$ and also again multiply by $P_{max}(i)$. In this way the combination of generator flexibility index can be simply derived as:

$$FLEX = \frac{\sum_{i=1}^{n} flex(i)}{\sum_{i=1}^{n} P_{max}(i)} \tag{8}$$

As the denominator of (4) is fixed for the system generation, it can be ignored in the system flexibility index calculation and the final generation flexibility index is written as:

$$FLEX = \sum_{i=1}^{n} flex(i) \tag{9}$$

Clearly, the up/down components of the system flexibility can be found in a combination of up/down components of the generation unit indices respectively.

$$FLEX\_UP = \sum_{i=1}^{n} Up\_Comp(i) \tag{10}$$

$$FLEX\_DN = \sum_{i=1}^{n} Dn\_Comp(i) \tag{11}$$

Where FLEX_UP and FLEX_DN are the up/down components of the system flexibility index.

6. Physical concept of the proposed index

Here the physical concept of (6), (10) and also (7), (11) is described. If we start from (10), it can be converted to:

$$\text{FLEX\_UP} = \sum_{i=1}^{n} \min\left(Rampup(i)\Delta t, \left(P_{max}(i) - P_g(i)\right)\right. \tag{12}$$

$$\text{FLEX\_UP} = \sum_{i=1}^{n} \min\left(Rampup(i)\Delta t + P_g(i), P_{max}(i)\right) - \sum_{i=1}^{n} P_g(i) \tag{13}$$

$$\text{FLEX\_UP} = \sum_{i=1}^{n} Up\_limit(i) - \sum_{i=1}^{n} P_g(i) \tag{14}$$

Up_limit(i) is the maximum admissible operation point for generator (i) at the time (t+Δt) (the solid/dotted red line in Fig. 6).

$$\text{FLEX\_UP} = \sum_{i=1}^{n} [Up\_limit(i) - P_g(i)] \tag{15}$$

So, the difference between Up_limit(i) and $P_g(i)$ can be considered as the upward generation reserve at the time (t+Δt).

$$\text{FLEX\_UP} = \sum_{i=1}^{n} Up\_Reserve(i) = UP\_RESERVE \tag{16}$$

If we follow the same procedure for (11), the downward reserve can be defined and (17) and (18) are modified for flexibility down component correspondingly.

$$\text{FLEX\_DN} = \sum_{i=1}^{n} [P_g(i) - Dn\_limit(i)] \tag{17}$$

$$\text{FLEX\_DN} = \sum_{i=1}^{n} Dn\_Reserve(i) = DN\_RESERVE \tag{18}$$

So, the up component of the system flexibility index can stand for the upward reserve of the generation system where the down component shows the downward reserve. It establishes a suitable relation between the flexibility index especially its two components and the accessible system reserve mainly upward/downward reserve.

7. Simulation

In this part 30 bus IEEE test system is used for simulation. This test system has six generation units. The generation data and daily load curve data are presented in Tables I & II in the appendix. The network loss coefficients are mentioned in the appendix too.

By the assumption all the units are in operation in the daily time interval, the unit commitment (economic load dispatch) solution for 24 hours is done. Fig. 7. shows the variation of total generation and the system generation maximum and minimum boundaries.

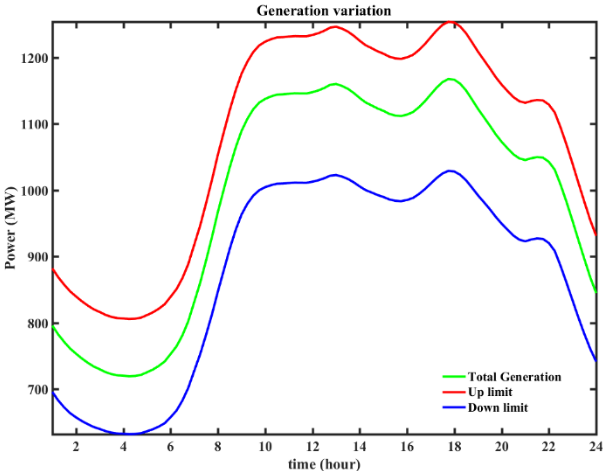

Fig.7. System generation, up/down constraints

When the total generation approaches the upper system generation limit, the system up reserve goes smaller. On the other hand, when the total generation approaches the downer system generation limit, less system down reserve is expected.

Fig. 8 shows the variation of the up/down components of the system flexibility index and also up/down system generation reserve. As can be seen, the up component of the flexibility index is the same as the generation up reserve. On the other hand, the down component of the flexibility index is the same as the system generation downward reserve. This is another reason for not dividing (5), (6) and (7) by $P_{max}(i)$ and also it needs to eliminate $\Sigma P_{max}(i)$ in (9), (10) and (11).

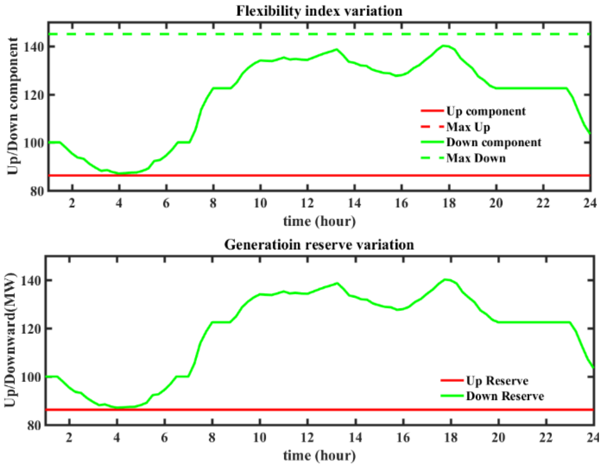

Fig.8. Up/down flexibility and system reserve

It is important the up component of the flexibility index lies on its maximum value with no change. This shows the maximum ability of the system generation exists to cope with the net load uncertainty and variability in the upward direction. But the down component varies in the daily time interval.

To show the variation of the flexibility index up component, the daily load curve is increased by 1.3125. Where the peak load is more than system generation capacity. So, it is expected load shedding in the peak load duration. Fig. 9. and Fig. 10. are similar to Fig. 7. and Fig. 8 correspondingly.

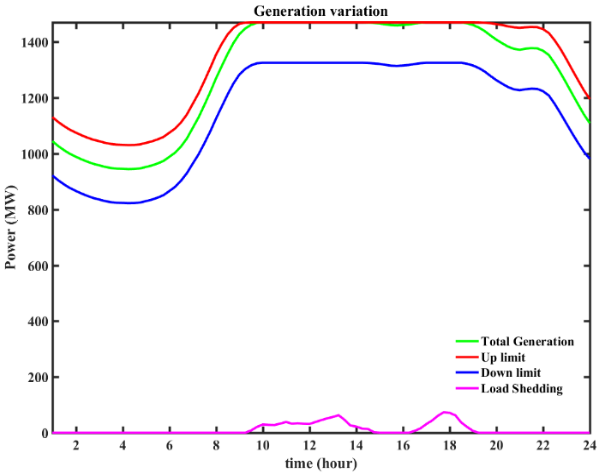

Fig. 9. System generation, up/down reserve and load shedding

As can be seen, the load shedding is forced into the system where the load plus system loss is more than system capacity. In contrast to the previous case, the downer component of the flexibility index lies in its maximum value in the large part of the load curve. But the up component goes down because of the considerable decrease in upward reserve and it lies zero when load shedding is forced into the system.

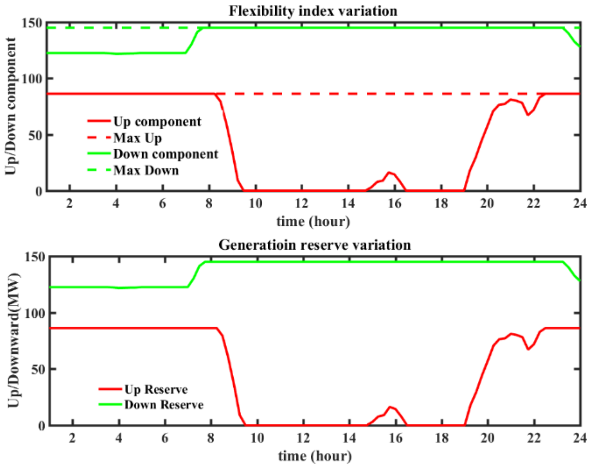

Fig. 10. Up/down flexibility and system reserve

8. Conclusion

Renewable energy integration is a main challenge for the power system flexibility, especially in the operational planning time zone. The current paper proposes an extension of the Normalized Flexibility Index (NFI) by considering the operation point of the generation unit/generation system as the main requirement of the flexibility index for use in the operation planning time zone. The proposed index has a direct relation to system generation reserve and can be converted to an economic value easily. This economic value can be combined with other system operation costs to find the optimal operation point considering the best level of the power system flexibility. It will be used to find the best penetration factor of the renewable energy sources or the best amount of the energy storage capacity in the power system integrated by the renewable sources such as large scale wind and solar farms.

9. References

[1] "Flexibility in the Power System – Danish and European experiences", Danish Energy Agency, 2015

[2] "Harnessing Variable Renewables", Tech. rep., International Energy Agency (IEA), Paris, 2011

[3] "Flexibility and Aggregation Requirements for their Interaction in the Market", Euroelectric, aisbl, Brussels, 2014

[4] H. Berahmandpour, Sh. Montasar Kouhsari, H. Rastegar, "A New Power System Operational Planning Flexibility Index Development", 33th Power system Conference (PSC), Tehran, Iran, 2018 (in Persian)

[5] E. Lannoye, D. Flynn and M. O'Malley, "Evaluation of Power System Flexibility" IEEE Transactions on Power Systems, vol. 27, no. 2, pp. 922-931, May 2012

[6] E. Lannoye, D. Flynn and M. O'Malley, "Power System Flexibility Assessment — State of the Art" 2012 IEEE Power and Energy Society General Meeting, pp. 1-6, 2012

[7] A. Ulbig, "Operational Flexibility in Electric Power Systems," A thesis submitted to the degree of Doctor of Science of ETH Zurich, 2014.

[8] A. A. Thatte, L. Xie, "A Metric and Market Construct of Inter-temporal Flexibility in Time-coupled Economic Dispatch", IEEE Trans. Power Syst. 31 (5), pp. 3437–3446, 2015.

[9] C. G Min and M. K. Kim, "Flexibility-based Evaluation of Variable Generation Acceptability in Korean Power System", Energies, vol. 10, no. 6, 825, 2017.

[10] C. G. Min, J. K. Park, D. Hur, and M. K. Kim, "A Risk Evaluation Method for Ramping Capability Shortage in Power Systems", Energy, vol. 113, pp. 1316–1324, 2016.

[11] C.-G. Min, "Analyzing the Impact of Variability and Uncertainty on Power System Flexibility", Appl. Sci. vol. 9, no. 3, pp. 1–13, 2019.

[12] A. Ulbig and G. Andersson, "On Operational Flexibility in Power Systems", 2012 IEEE Power and Energy Society General Meeting, pp. 1-8, 2012

[13] J. Ma, V. Silva, R. Belhomme, D. S. Kirschen and L. F. Ochoa, "Exploring the Use of Flexibility Indices in Low Carbon Power Systems", 2012 3rd IEEE PES Innovative Smart Grid Technologies Europe (ISGT Europe), pp. 1–5, 2012

10. Appendix

a):Test system data

Table I. Generation data

Unit No.	P_{min}(MW)	P_{max}(MW)	Rampup(MW/h)	Rampdn(MW/h)	a ($/MW^2)	b ($/MW)	c ($)
1	100	500	120	80	0.0070	7.0000	240
2	50	200	90	50	0.0095	10.0000	200
3	80	300	100	65	0.0090	8.5000	220
4	50	150	90	50	0.0090	11.0000	200
5	50	200	90	50	0.0080	10.5000	220
6	50	120	90	50	0.0075	12.0000	190

Table II. Daily load data (MW)

Time	**Load**	Time	**Load**	Time	**Load**	Time	**Load**	Time	**Load**	Time	**Load**	Time	**Load**	Time	**Load**
00.00	**839.2**	03.00	**722.4**	06.00	**756.4**	09.00	**1113.1**	12.00	**1149.0**	15.00	**1114.1**	18.00	**1167.3**	21.00	**1041.8**
00.15	**826.2**	03.15	**723.6**	06.15	**782.4**	09.15	**1126.1**	12.15	**1154.3**	15.15	**1113.1**	18.15	**1147.8**	21.15	**1046.4**
00.30	**806.7**	03.30	**720.2**	06.30	**796.1**	09.30	**1135.8**	12.30	**1159.1**	15.30	**1107.6**	18.30	**1135.8**	21.30	**1063.3**
00.45	**793.5**	03.45	**718.3**	06.45	**822.9**	09.45	**1142.5**	12.45	**1163.0**	15.45	**1109.0**	18.45	**1124.9**	21.45	**1057.0**
01.00	**780.7**	04.00	**718.8**	07.00	**854.7**	10.00	**1141.6**	13.00	**1167.8**	16.00	**1114.1**	19.00	**1106.4**	22.00	**1035.6**
01.15	**770.3**	04.15	**719.5**	07.15	**900.4**	10.15	**1141.1**	13.15	**1155.3**	16.15	**1124.9**	19.15	**1097.5**	22.15	**1010.5**
01.30	**759.8**	04.30	**719.8**	07.30	**926.4**	10.30	**1144.9**	13.30	**1139.9**	16.30	**1131.9**	19.30	**1084.9**	22.30	**986.7**
01.45	**750.6**	04.45	**722.4**	07.45	**964.5**	10.45	**1149.3**	13.45	**1137.0**	16.45	**1143.5**	19.45	**1072.7**	22.45	**948.4**
02.00	**743.9**	05.00	**726.3**	08.00	**999.9**	11.00	**1144.7**	14.00	**1131.4**	17.00	**1153.4**	20.00	**1058.9**	23.00	**927.2**
02.15	**741.9**	05.15	**738.3**	08.15	**1043.5**	11.15	**1145.9**	14.15	**1130.2**	17.15	**1166.4**	20.15	**1049.8**	23.15	**895.4**
02.30	**733.7**	05.30	**740.0**	08.30	**1070.0**	11.30	**1144.4**	14.30	**1121.8**	17.30	**1176.0**	20.30	**1048.6**	23.30	**865.3**
02.45	**727.7**	05.45	**747.0**	08.45	**1092.4**	11.45	**1144.0**	14.45	**1117.7**	17.45	**1174.3**	20.45	**1039.4**	23.45	**844.8**

b) Loss coefficient matrix

Loss=$P^tBP+B_0P+B_{00}$

b-1) B matrix

$$\begin{matrix} 0.0017 & 0.0012 & 0.0007 & -0.0001 & -0.0005 & -0.0002 \\ 0.0012 & 0.0014 & 0.0009 & 0.0001 & -0.0006 & -0.0001 \\ 0.0007 & 0.0009 & 0.0031 & 0.0000 & -0.0010 & -0.0006 \\ -0.0001 & 0.0001 & 0.0000 & 0.0024 & -0.0006 & -0.0008 \\ -0.0005 & -0.0006 & -0.0010 & -0.0006 & 0.0129 & -0.0002 \\ -0.0002 & -0.0001 & -0.0006 & -0.0008 & -0.0002 & 0.0150 \end{matrix}$$

b-2) B_0 vector

$$\begin{matrix} -0.3908 & -0.1297 & 0.7047 & 0.0591 & 0.2161 & -0.6635 \end{matrix}$$

b-3) B_{00} constant

$B_{00}=0.056$