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Observer-based Event-Triggered Guaranteed Cost Leader-Following Consensus Control for Heterogeneous Uncertain Nonlinear Fractional-Order Multi-Agent Systems

Mehdi Asaiyan[1], Mahdi Pourgholi[1,*]

[1] *Faculty of Electrical Engineering, Shahid Beheshti University, Tehran, Iran*

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ABSTRACT

Recently, the guaranteed cost consensus problem of multi-agent systems has attracted the attention of researchers. This paper tackles the challenge of event-triggered guaranteed cost leader-following consensus in heterogeneous uncertain nonlinear fractional-order multi-agent systems employing observers. The agents have different fractional-order dynamics coupled with uncertainties in their state, input, and output. To optimize communication resources, the paper introduces an event-triggered strategy, ensuring that updates to the control protocol occur only upon the satisfaction of the triggering condition. Leveraging this strategy and applying the fractional Lyapunov direct method, the problem is formulated. To obtain control and observer gains, a systematic approach algorithm is proposed using Linear Matrix Inequalities (LMI), with corresponding criteria established to guarantee guaranteed cost consensus. The effectiveness of the proposed method is validated through a numerical simulation, with comprehensive results presented. This research not only addresses a complex problem in multi-agent systems but also contributes a practical and resource-efficient solution, showcasing its potential applicability in real-world scenarios.

1. Introduction

In multi-agent systems (MASs), multiple simple agents cooperate to perform a large-scale complicated task. In the last decade, due to the wide practical applications of MAS in various fields including unmanned aerial vehicle formation [1], sensor network synchronization [2], power grid synchronization control [3], and intelligent transportation [4], there have been many studies on the coordination of MASs. The consensus problem as one of the cooperative control problems indicates that all agents reach a common final state through the exchange of local information with their neighbors. Specifically, the leader-following consensus problem arises when there is a leading agent to provide the agreement state. Therefore, the control objective of MAS can be realized by controlling only the leader, which not only significantly simplifies the analysis and design of MAS but also helps to save energy and reduce control costs [5].

Researchers have designed many controllers, including optimal and adaptive control [6, 7], distributed impulsive control [8], and fuzzy control [9], to achieve leader-follower consensus of homogeneous MASs where all agents have the same dynamics. However, in many cases, the agents are heterogeneous, which means that their dynamics and even the dimensions of their state space are different.

Physical systems are subject to various model uncertainties and various practical nonlinear phenomena, which may originate from changes in system parameters or modeling errors. Failure to properly deal with these phenomena can reduce the closed-loop performance of systems or may even make systems unstable. Therefore, the robust control of MASs has been the target of many researchers in recent years [10-12]. In [11] the robust control problem of linear homogeneous MASs with different norm-bounded uncertainties is studied. Further by using a distributed observer-based protocol, the robust

* Corresponding author
E-mail address: m_pourgholi@sbu.ac.ir
https://orcid.org/0000-0002-9679-1067
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control problem synthesized with transient performance is studied in [12] .

In addition, fractional-order models which are an extension of integer-order models, can more accurately describe systems due to their superior performance in depicting the memory and intrinsic properties of different types of materials and processes [13, 14], for example, motion of multiple agents in viscoelastic materials or macromolecule fluids.

In recent years, many research works have addressed the issue of consensus control of fractional-order MASs (FOMASs) [15-22]. Authors in [15] based on the linear matrix inequalities, proposed a distributed state feedback consensus protocol for consensus of heterogeneous FOMAS. In [16] Gong studied a distributed leader-following of heterogeneous nonlinear FOMASs with an unknown leader. In [17, 18] Gong et al. investigated adaptive robust leader-following consensus control for uncertain nonlinear FOMASs. In [19] Gong et al. proposed a distributed robust consensus control of heterogeneous FOMASs. In [20] Gong et al. investigated the output feedback consensus control problem for a class of nonlinear FOMASs. In [21] Gong et al. investigated robust adaptive fault-tolerant consensus control for uncertain nonlinear FOMASs. In [22] Wen et al. proposed an observer and virtual exo-system based output consensus of leader-following heterogeneous nonlinear FOMASs.

In all the above aforementioned works, the consensus problem of FOMASs are obtained while transmission information between agents are continuous which is difficult to implement in practice. Periodic sampling is a good method to transmission information between agents. However, when the sampling period is very small, it can lead to a loss of communication resources. Thus, time-triggered sampling is used instead of event-triggered sampling in recent works. Many research works have been done to apply event-triggered strategies to the consensus problem of MASs [23-27]. Authors in [23] investigated the event-triggered leader-following consensus problem for MASs with semi-Markov switching topologies.

In [24] Li et al. proposed a dynamic event-triggered control for heterogeneous leader-following consensus of MASs based on input to state stability. In [25] Yang et al. studied the leader-following output consensus problem of heterogeneous linear MASs, where followers are subject to parameter uncertainties. In [26] Ren et al. investigated the consensus of general linear FOMASs by distributed event-triggered strategy. In [27] Hu et al. proposed the event-triggered leader-following consensus for FOMASs.

In practical uses of MASs, agents can only have limited energy resources to perform certain tasks such as cognition, transmission information, and movement. Recently, the guaranteed cost consensus (GCC) problem of MASs has abundantly attracted the attention of researchers [28-32]. In [28] Wang et al. investigated the GCC control for MASs with fixed interaction topologies. In [29] Wang et al. investigated the guaranteed performance consensus for the Lipschitz class of nonlinear MASs. In [30] Luo et al. proposed event-triggered GCC for uncertain nonlinear MASs. In [31] Luo et al. proposed observer-based event-triggered GCC control for second-order MASs. In [32] Tian et al. investigated the leaderless GCC for uncertain, delayed nonlinear FOMASs.

Motivated by the above discussion, the event-triggered leader-following GCC for heterogeneous uncertain nonlinear FOMASs based on observers proposed in this paper.

The main contributions of this paper can be summarized as follows:

- An observer-based output feedback control for event-triggered consensus of nonlinear FOMASs with state, input, and output uncertainty is proposed.
- GCC by the event-triggered strategy for FOMASs is obtained.
- To obtain control and observer gains, a systematic approach using linear matrix inequality (LMI) algorithm is proposed.

The rest of the article is organized as follows: In Section 2 essential concepts and useful lemmas are provided. Section 3 presents the main theorems. A numerical example is provided in Section 4 and finally, in Section 5 conclusion remarks are given.

2. Preliminary and Problem Formulation

A. Notations

In this paper, $\|.\|$ and $\otimes$ represent the Euclidean norm and the Kronecker product, respectively. $I_N \in \mathbb{R}^{n\times n}$ is an identity matrix. $diag\{*\}$ and $blockdiag\{*\}$ denote the diagonal matrix and the block diagonal matrix, respectively. The statement $A > 0(\geq 0)$ and $A < 0(\leq 0)$ represent symmetric positive and negative definite (semi-definite) matrices, respectively. The matrices A^{-1} and A^T denote inverse and the transpose of A, respectively. $\bar{N} \triangleq \{1,2,\ldots,N\}$.

B. Graph theory

Consider a MAS composed of N follower agents and a leader. The interaction among N followers can be denoted by a weighted digraph $\mathcal{G} = (\mathcal{V},\mathcal{E})$, where $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ represent the set of nodes and the set of directed edges of $\mathcal{G}$, respectively. An edge $\varepsilon_{ji} = (v_j, v_i) \in \mathcal{E}$ means that agent j can transmit information to agent i and they are called the parent node and the child node, respectively. $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{N\times N}$ denotes the weighted adjacency matrix where $a_{ij} > 0$ if $\varepsilon_{ji} \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Besides $a_{ii} = 0$ for $i \in \bar{N}$. The diagonal matrix $\mathcal{D} = diag\{d_1, d_2, \ldots, d_N\}$ is the degree matrix where the elements are defined by $d_i = \sum_{j=1}^{N} a_{ij}$ and the Laplacian matrix of the weighted digraph $\mathcal{G}$ is defined as $\mathcal{L} = \mathcal{D} - \mathcal{A} = [l_{ij}] \in \mathbb{R}^{N\times N}$, i.e. $l_{ii} = \sum_{j=1}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$ for $i \neq j$. Letting node 0 be associated with the leader, the communication among all followers and the leader can be described by a new directed graph $\bar{\mathcal{G}} = (\bar{\mathcal{V}}, \bar{\mathcal{E}})$, where $\bar{\mathcal{V}} = \mathcal{V} \cup \{v_0\}$ and $\bar{\mathcal{E}} \subseteq \bar{\mathcal{V}} \times \bar{\mathcal{V}}$. The diagonal matrix $\mathcal{B} = diag\{b_1, b_2, \ldots, b_N\}$ denote the weights of the directed edges from leader to followers in the digraph $\bar{\mathcal{G}}$. If $b_i > 0$, there exists a directed edge from the leader to the follower i and $b_i = 0$ otherwise. A digraph contains a directed spanning tree if there exists a node called root, which has no parent node and this root node has directed paths to all other nodes in this graph.

Lemma 1 [22]: The directed graph of all agents has a directed spanning tree with the leader rooted if all eigenvalues of matrix $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{L}} + \boldsymbol{\mathcal{B}}$ have positive real parts and vice versa.

C. Fractional-order operators

The Caputo and Riemann-Liouville are two well-known fractional-order derivatives, since he initial conditions in Caputo fractional-order derivative is as same as the integer-order differential equations, thus to model the FOMASs we use the Caputo derivative in this paper.

Definition 1 [33] (Riemann-Liouville Integral): The Riemann-Liouville fractional-order integral of function $\boldsymbol{x(t)}$ of order $\boldsymbol{\alpha}$ is defined as:

$$ {}_{t_0}^{RL}I_t^{\alpha} x(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-\tau)^{\alpha-1} x(\tau) d\tau \tag{1}$$

where $\alpha \in (n-1, n], n \in \mathbb{Z}^+$ and $\Gamma(s) = \int_0^{\infty} t^{s-1} e^{-t} dt$ is the Gamma function. For convenience, we use the notion $I^{\alpha} x(t)$ to denote ${}_{t_0}^{RL}I_t^{\alpha} x(t)$ later.

Definition 2 [33] (Caputo Derivative): The Caputo fractional-order derivative of function $x(t)$ of order α defined as:

$$ {}_{t_0}^{C}D_t^{\alpha} x(t) = I^{n-\alpha} x^{(n)}(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} \frac{x^{(n)}(\tau)}{(t-\tau)^{\alpha+1-n}} d\tau \tag{2}$$

where n is a positive integer satisfying $n - 1 \leq \alpha < n$. For convenience, we use the notion $D^{\alpha} x(t)$ to denote ${}_{t_0}^{C}D_t^{\alpha} x(t)$ later.

Lemma 2 [34] (Fractional Lyapunov direct method): Let $\boldsymbol{x = 0}$ be an equilibrium point for the nonautonomous fractional order system $\boldsymbol{D^{\alpha} x(t) = f(t, x)}$ where $\boldsymbol{a} \in (\boldsymbol{0}, \boldsymbol{1}]$, $\boldsymbol{f}: [\boldsymbol{t_0}, \infty) \times \boldsymbol{\Omega} \to \mathbb{R}^{\boldsymbol{n}}$ is piecewise continuous in $\boldsymbol{t}$ and locally Lipschitz in $\boldsymbol{x}$ on $[\boldsymbol{t_0}, \infty) \times \boldsymbol{\Omega}$ and $\boldsymbol{\Omega} \in \mathbb{R}^{\boldsymbol{n}}$ is a domain that contains the origin $\boldsymbol{x = 0}$. Assume that there exists a Lyapunov function $\boldsymbol{V(t, x(t))}$ and class $\boldsymbol{\mathcal{K}}$ functions $\boldsymbol{\alpha_i (i = 1, 2, 3)}$ satisfying:

$$\alpha_1(\|x\|) \leq V(t, x) \leq \alpha_2(\|x\|) \tag{3}$$

$$D^{\alpha} V(t, x) \leq -\alpha_3(\|x\|) \tag{4}$$

Then the equilibrium point is asymptotically stable.

Lemma 3 [32]: For a differentiable vector $\boldsymbol{x(t)} \in \mathbb{R}^{\boldsymbol{n}}$, a symmetric positive-definite $\boldsymbol{P}$ and $\boldsymbol{\alpha} \in (\boldsymbol{0}, \boldsymbol{1}]$ the following inequality is true:

$$D^{\alpha}\left(x^T(t) P x(t)\right) \leq D^{\alpha} x^T(t) P x(t) + x^t(t) P D^{\alpha} x(t) \tag{5}$$

D. Problem description

Consider a heterogeneous uncertain nonlinear FOMAS with N agents and a leader. The dynamics of agent i is described by:

$$\begin{cases} D^{\alpha} x_i(t) = \left(A_i + \Delta A_i(t)\right) x_i(t) + \left(B_i + \Delta B_i(t)\right) u_i(t) + f_i\left(x_i(t)\right) \\ y_i(t) = \left(C_i + \Delta C_i(t)\right) x_i(t) \end{cases} \tag{6}$$
$$i \in \bar{N} \cup \{0\}$$

where $0 < \alpha \leq 1$, $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^{m_i}$ and $y_i \in \mathbb{R}^{o_i}$ are the pseudo-state, control input and measurable output vectors of agent i, respectively. The matrices $A_i \in \mathbb{R}^{n \times n}$, $B_i \in \mathbb{R}^{n \times m_i}$ and $C_i \in \mathbb{R}^{o_i \times n}$ are nominal parts and $\Delta A_i(t)$, $\Delta B_i(t)$ and $\Delta C_i(t)$ represent time-varying parameter uncertainties and $f_i\left(x_i(t)\right)$ represents the nonlinear dynamic function.

Definition 3: The MAS represented in (6) said to achieve leader-following consensus if $\boldsymbol{\lim_{t \to \infty} \|x_i(t) - x_0(t)\| = 0}$, $\forall \boldsymbol{i} \in \bar{\boldsymbol{N}}$.

Assumption 1: The pairs $(\boldsymbol{A_i}, \boldsymbol{B_i})$ are stabilizable and the pairs $(\boldsymbol{C_i}, \boldsymbol{A_i})$ are detectable $\forall \boldsymbol{i} \in \bar{\boldsymbol{N}} \cup \{\boldsymbol{0}\}$.

Assumption 2: Time-varying parameter uncertainties can formulated as:

$$\begin{gathered} [\Delta A_i(t) \quad \Delta B_i(t)] = M_i H_i(t) [\tilde{A}_i \quad \tilde{B}_i], \\ \Delta C_i(t) = N_i H_i(t) \tilde{C}_i \end{gathered} \tag{7}$$

where M_i, N_i, $\tilde{A}_i$, $\tilde{B}_i$ and $\tilde{C}_i$ are real constant matrices of appropriate dimensions, and $H_i(t)$ is the real unknown time-varying matrix which satisfy $H_i^T(t) H_i(t) \leq I$ $\forall i \in \bar{N} \cup \{0\}$.

Assumption 3: The nonlinear functions $\boldsymbol{f_i(x_i(t))}: \mathbb{R}^{\boldsymbol{n}} \to \mathbb{R}^{\boldsymbol{n}}$ are continuous functions that satisfy the following Lipschitz conditions:

$$\begin{gathered} \|f_i(x) - f_i(y)\| \leq \theta_i \|x - y\|, \\ \|f_i(x)\| \leq \theta_i \|x\| \end{gathered} \tag{8}$$

$$\|f_i(x) - f_0(y)\| \leq \sigma_i \|x - y\| \tag{9}$$

where $\boldsymbol{\theta_i}$ and $\boldsymbol{\sigma_i}$ are known constants $\forall \boldsymbol{i} \in \bar{\boldsymbol{N}} \cup \{\boldsymbol{0}\}$.

Assumption 4: The interaction topology of all agents contains a directed spanning tree with the leader as the root.

3. Observers and virtual systems

This paper allows systems pseudo-state to be immeasurable. For each agent, we consider an observer that is described as follows:

$$\begin{gathered} D^{\alpha} \hat{x}_i(t) = A_i \hat{x}_i(t) + B_i u_i(t) + f_i\left(\hat{x}_i(t)\right) \\ - E_i\left(y_i(t) - C_i \hat{x}_i(t)\right), \\ i \in \bar{N} \cup \{0\} \end{gathered} \tag{10}$$

where $\hat{x}_i(t) \in \mathbb{R}^n$ denotes estimate of the pseudo-state $x_i(t)$ and $E_i \in \mathbb{R}^{n \times o_i}$ denotes the observer gain matrix and we define observer error as $\eta_i(t) = x_i(t) - \hat{x}_i(t)$.
Moreover, for each follower we consider a virtual system that is described as follows:

$$\begin{gathered} D^{\alpha} \check{x}_i(t) = (A_0 + B_0 K_0) \check{x}_i(t) + f_0\left(\check{x}_i(t)\right) \\ + F_i v_i(t), \qquad i \in \bar{N} \end{gathered} \tag{11}$$

where $\check{x}_i(t) \in \mathbb{R}^n$ denotes virtual pseudo-state of agent i and $F_i \in \mathbb{R}^{n \times n}$ denotes the consensus gain matrix and we define virtual consensus error as $\zeta_i(t) = \check{x}_i(t) - \hat{x}_0(t)$.
Control inputs is designed as:

$$\begin{gathered} u_0(t) = K_0 \hat{x}_0(t), \qquad u_i(t) \\ = K_i \hat{x}_i(t) \\ + W_i\left(\hat{x}_i(t) - \check{x}_i(t)\right), \quad i \in \bar{N} \end{gathered} \tag{12}$$

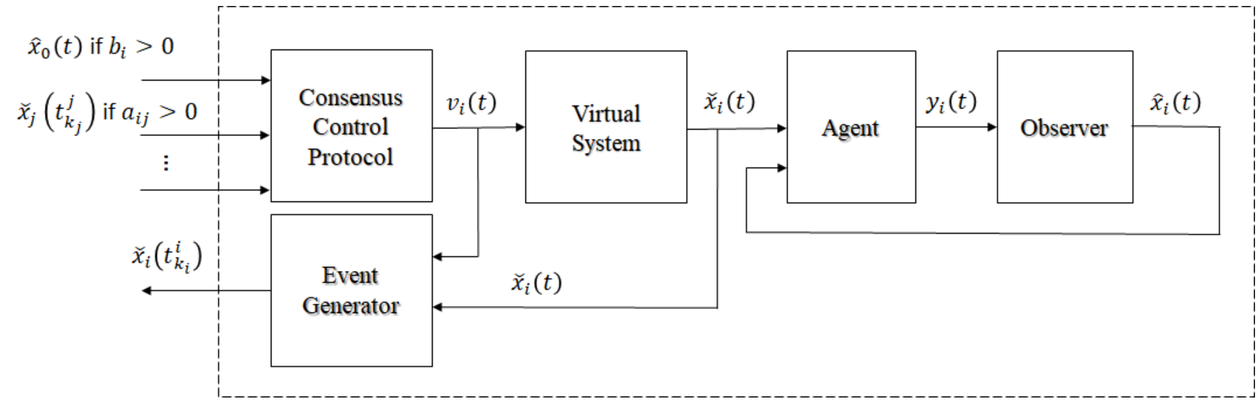

Figure 1: Observer-based event-triggered control scheme for the follower i.

where $K_i \in \mathbb{R}^{m_i \times n}$ and $W_i \in \mathbb{R}^{m_i \times n}$ denotes the pseudo-state feedback gain matrix and consensus feedback gain matrix.

4. Event-triggered control strategy

The approximation error between the current instant and the last event instant for virtual pseudo-state of agent i defined as:

$$e_i(t) = \check{x}_i\left(t_{k_i}^i\right) - \check{x}_i(t), \quad \forall t \in \left[t_{k_i}^i, t_{k_i+1}^i\right), \quad i \in \bar{N} \tag{13}$$

The event-triggered consensus control protocol described as:

$$v_i(t) = \sum_{j=1}^N a_{ij}\left(\check{x}_i\left(t_{k_i}^i\right) - \check{x}_j\left(t_{k_j}^j\right)\right) + b_i\left(\check{x}_i\left(t_{k_i}^i\right) - \hat{x}_0(t)\right), \; i \in \bar{N} \tag{14}$$

where $\check{x}_i\left(t_{k_i}^i\right)$ is the virtual pseudo-state of agent i at the k_i time event-triggered.

For agent i the event-triggered strategy can be designed as follows:

$$\begin{aligned} t_1^i &= 0, \\ t_{k_i+1}^i &= \inf\left\{t > t_{k_i}^i \middle| e_i^T(t) T_i e_i(t) > \rho v_i^T(t) Q_i v_i(t)\right\} \\ i &\in \bar{N} \end{aligned} \tag{15}$$

here the time sequence $t_{k_i}^i$ represents the event-triggered for agent i, the adjacent sampling instants represent by $t_{k_i}^i$ and $t_{k_i+1}^i$. $T_i = T_i^T$ and $Q_i = Q_i^T$ are positive-definite matrices, and event-triggered threshold is shown by a scalar value $\rho \in [0,1]$. When $t = t_{k_i}^i$, to update the control protocol, the new sampled virtual pseudo-state is conducted to the sub-controller.

The observer-based event-triggered control scheme for follower i is shown in Figure 1.

G. Guaranteed cost

The guaranteed cost functions related to the leader's system, virtual systems, and follower's systems are defined as follows:

$$J_{x_0} = \lim_{t\to\infty}\left[I^\alpha\left(j_{\eta_0}(t) + j_{u_0}(t)\right)\right] \tag{16}$$

$$J_{x_i} = \lim_{t\to\infty}\left[I^\alpha\left(j_{\eta_i}(t) + j_{\varphi_i}(t) + j_{u_i}(t)\right)\right], \; i \in \bar{N} \tag{17}$$

$$J_{\check{x}} = \lim_{t\to\infty}\left[I^\alpha\left(j_\zeta(t) + j_v(t)\right)\right] \tag{18}$$

$$j_{\eta_i}(t) = \eta_i^T(t) G_{\eta_i} \eta_i(t), \; i \in \bar{N} \cup \{0\} \tag{19}$$

$$j_{\varphi_i}(t) = \varphi_i^T(t) G_{\varphi_i} \varphi_i(t), \; i \in \bar{N} \tag{20}$$

$$j_{u_i}(t) = u_i^T(t) G_{u_i} u_i(t), \; i \in \bar{N} \cup \{0\} \tag{21}$$

$$j_\zeta(t) = \sum_{i=1}^N\left\{\zeta_i^T(t) G_{\zeta_i} \zeta_i(t)\right\} \tag{22}$$

$$j_v(t) = \sum_{i=1}^N\left\{v_i^T(t) F_i^T G_{v_i} F_i v_i(t)\right\} \tag{23}$$

where $\eta_i(t)$, $\varphi_i(t) = x_i(t) - \check{x}_i(t)$, $u_i(t)$, $\zeta_i(t)$ and $v_i(t)$ are observer error, consensus error, control input, virtual consensus error and the event-triggered consensus control protocol of agent i, respectively and $G_{\eta_i} > 0$, $G_{\varphi_i} > 0$, $G_{u_i} > 0$, $G_{\zeta_i} > 0$ and $G_{v_i} > 0$ are given symmetric matrices.

Definition 4: The MAS represented in (6) with cost functions (16), (17) and (18) is said to achieve leader-following GCC if there exist positive scalars $\boldsymbol{J}_{\boldsymbol{x}_0}^*, \boldsymbol{J}_{\boldsymbol{x}_i}^*$ and $\boldsymbol{J}_{\check{\boldsymbol{x}}}^*$ such that the leader-following consensus is achieved and cost functions are satisfies the inequalities $\boldsymbol{J}_{\boldsymbol{x}_0} < \boldsymbol{J}_{\boldsymbol{x}_0}^*, \boldsymbol{J}_{\boldsymbol{x}_i} < \boldsymbol{J}_{\boldsymbol{x}_i}^*$ and $\boldsymbol{J}_{\check{\boldsymbol{x}}} < \boldsymbol{J}_{\check{\boldsymbol{x}}}^*$ where $\boldsymbol{J}_{\boldsymbol{x}_0}^*, \boldsymbol{J}_{\boldsymbol{x}_i}^*$ and $\boldsymbol{J}_{\check{\boldsymbol{x}}}^*$ are called guaranteed cost upper bound.

Lemma 4 [35]: For any real vector with the appropriate dimension x and y, the following inequality is true:

$$x^T y + y^T x \le \beta x^T x + \beta^{-1} y^T y \tag{24}$$

where $\boldsymbol{\beta}$ is a positive number.

Lemma 5 [36] (Congruence Transformation): For a symmetric matrix A and a invertible matrix T, $T^T A T$ is negative definite if and only if A is negative definite.

Lemma 6 [36] (Schur Complement): For a symmetric matrix A and a symmetric invertible matrix $\mathcal{Q}$, $\begin{bmatrix} A & \mathcal{P} \\ \mathcal{P}^T & \mathcal{Q} \end{bmatrix}$ is negative definite if and only if $\mathcal{Q}$ and $A - \mathcal{P}\mathcal{Q}^{-1}\mathcal{P}^T$ are negative definite.

3. Main results

To achieve leader-following consensus according to mentioned strategy, first we simplified the consensus error as follows:

$$\begin{aligned} \delta_i(t) &= x_i(t) - x_0(t) \\ &= x_i(t) - \hat{x}_0(t) - \eta_0(t) \\ &= x_i(t) - \check{x}_i(t) + \zeta_i(t) - \eta_0(t) \end{aligned} \tag{25}$$

thus if $\eta_0(t)$, $\zeta_i(t)$ and $\varphi_i(t) = x_i(t) - \check{x}_i(t)$ are stable then $\delta_i(t)$ is stable.
In three steps we achieve stability of $\eta_0(t)$, $\zeta_i(t)$ and $\varphi_i(t) = x_i(t) - \check{x}_i(t)$, respectively.

Step 1: Stability of leader's system and observer error

In this step first we determine K_0 considering to stability of the leader's system and desirable dynamics then we determine E_0 considering to dynamics of leader's system and stability of observer error of leader.

Theorem 1: Considering Assumption 1-3 are met and $\boldsymbol{\eta_0(t) = 0}$. For leader's system $(\boldsymbol{6})$ if there exit a matrix $\boldsymbol{P_{x_0} = P_{x_0}^T > 0}$, a matrix $\boldsymbol{Y_{x_0}}$ and positive constants $\boldsymbol{\alpha_{x_0}}$, $\boldsymbol{\beta_{x_0}}$ and $\boldsymbol{\omega_{x_0}}$ satisfying the following condition:

$$\Pi_{x_0} = \begin{bmatrix} \Pi_{x_{0}11} & \wp_{x_{0}1} \\ * & q_{x_{0}1} \end{bmatrix} < 0 \tag{26}$$

where

$$\Pi_{x_{0}11} = X_{x_0}A_0^T + A_0X_{x_0} + Y_{x_0}^TB_0^T + B_0Y_{x_0} + (\alpha_{x_0} + \beta_{x_0})M_0M_0^T + \omega_{x_0}I_n$$

$$\wp_{x_{0}1} = \begin{bmatrix} X_{x_0}\tilde{A}_0^T & Y_{x_0}^T\tilde{B}_0^T & X_{x_0} & Y_{x_0}^T \end{bmatrix}$$

$$q_{x_{0}1} = -blockdiag(\alpha_{x_0}I_n, \beta_{x_0}I_n, \omega_{x_0}\theta_0^{-2}I_n, G_{u_0}^{-1})$$

Then, under control input (12) with $K_0 = Y_{x_0}X_{x_0}^{-1}$, the leader's system is asymptotically stable with guaranteed cost upper bound $J_{x_0}^* = x_0^T(0)P_{x_0}x_0(0)$ for the cost function $J_{x_0} = \lim_{t\to\infty}[I^\alpha j_{u_o}(t)]$.

Proof: Apply the control input (12) to the leader's system (6) yields:

$$D^\alpha x_0(t) = \big(A_0 + \Delta A_0(t) + (B_0 + \Delta B_0(t))K_0\big)x_0(t) - (B_0 + \Delta B_0(t))K_0\eta_0(t) + f_0(x_0(t)) \tag{27}$$

Consider following Lyapunov candidate function:

$$V_{x_0}(t) = x_0^T(t)P_{x_0}x_0(t) \tag{28}$$

where P_{x_0} is an unknown symmetric positive-definite matrix.
Taking the α-order derivative and using Lemma 3 yields:

$$D^\alpha V_{x_0}(t) \le \Big(\big(A_0 + \Delta A_0(t) + (B_0 + \Delta B_0(t))K_0\big)x_0(t) - (B_0 + \Delta B_0(t))K_0\eta_0(t) + f_0(x_0(t))\Big)^T P_{x_0}x_0(t) + x_0^T(t)P_{x_0}\Big(\big(A_0 + \Delta A_0(t) + (B_0 + \Delta B_0(t))K_0\big)x_0(t) - (B_0 + \Delta B_0(t))K_0\eta_0(t) + f_0(x_0(t))\Big) \tag{29}$$

Assuming $\eta_0(t) = 0$ and using Lemma 4 yields:

$$D^\alpha V_{x_0}(t) \le x_0^T(t)\Sigma_{x_0}x_0(t) \tag{30}$$

where

$$\Sigma_{x_0} = (A_0 + B_0K_0)^TP_{x_0} + P_{x_0}(A_0 + B_0K_0) + \alpha_{x_0}^{-1}\tilde{A}_0^T\tilde{A}_0 + \beta_{x_0}^{-1}K_0^T\tilde{B}_0^T\tilde{B}_0K_0 + \omega_{x_0}^{-1}\theta_0^2I_n + \omega_{x_0}P_{x_0}P_{x_0} + (\alpha_{x_0} + \beta_{x_0})P_{x_0}M_0M_0^TP_{x_0}$$

According to fractional Lyapunov direct method, leader's system is asymptotically stable If $\Sigma_{x_0} < 0$.
Furthermore, we consider following cost function:

$$J_{x_0} = \lim_{t\to\infty}[I^\alpha j_{u_o}(t)] \tag{31}$$

$$j_{u_0}(t) = u_0^T(t)G_{u_0}u_0(t) = x_0^T(t)K_0^TG_{u_0}K_0x_0(t) \tag{32}$$

If $D^\alpha V_{x_0}(t) + j_{u_0}(t) < 0$:

$$j_{u_0}(t) < -D^\alpha V_{x_0}(t) \tag{33}$$

Then for $t \in [0, \infty)$, the α-order integrating both sides yields:

$$I^\alpha j_{u_o}(t) < V_{x_0}(0) - V_{x_0}(t) \tag{34}$$

Since $D^\alpha V_{x_0}(t) < 0$, $\lim_{t\to\infty} V_{x_0}(t) = 0$, therefore:

$$J_{x_0} = \lim_{t\to\infty}[I^\alpha j_{u_o}(t)] < V_{x_0}(0) = x_0^T(0)P_{x_0}x_0(0) = J_{x_0}^* \tag{35}$$

So, the upper bound $J_{x_0}^*$ of the quadratic guaranteed cost function can be obtained.

$$D^\alpha V_{x_0}(t) + j_{u_0}(t) \le x_0^T(t)\hat{\Sigma}_{x_0}x_0(t) \tag{36}$$

where

$$\hat{\Sigma}_{x_0} = (A_0 + B_0K_0)^TP_{x_0} + P_{x_0}(A_0 + B_0K_0) + \alpha_{x_0}^{-1}\tilde{A}_0^T\tilde{A}_0 + \beta_{x_0}^{-1}K_0^T\tilde{B}_0^T\tilde{B}_0K_0 + \omega_{x_0}^{-1}\theta_0^2I_n + \omega_{x_0}P_{x_0}P_{x_0} + (\alpha_{x_0} + \beta_{x_0})P_{x_0}M_0M_0^TP_{x_0} + K_0^TG_{u_0}K_0$$

For linearization the inequality $\hat{\Sigma}_{x_0} < 0$ by using Lemma 5 and pre- and post-multiplying both sides of the inequality by $X_{x_0} = P_{x_0}^{-1}$ and let $Y_{x_0} = K_0P_{x_0}^{-1}$, the following obtain:

$$X_{x_0}\hat{\Sigma}_{x_0}X_{x_0} = X_{x_0}A_0^T + A_0X_{x_0} + Y_{x_0}^TB_0^T + B_0Y_{x_0} + \alpha_{x_0}^{-1}X_{x_0}\tilde{A}_0^T\tilde{A}_0X_{x_0} + \beta_{x_0}^{-1}Y_{x_0}^T\tilde{B}_0^T\tilde{B}_0Y_{x_0} + \omega_{x_0}^{-1}\theta_0^2X_{x_0}X_{x_0} + \omega_{x_0}I_n + (\alpha_{x_0} + \beta_{x_0})M_0M_0^T + Y_{x_0}^TG_{u_0}Y_{x_0} < 0 \tag{37}$$

By employing Lemma 6, we can obtain (26) and this completes the proof.

Remark 1: In solving LMI in Theorem 1, to reach the desirable dynamics of pseudo-state like $\boldsymbol{D^\alpha x(t) = A_d x(t)}$, we can use objective $\boldsymbol{(A_0 - A_d)X_{x_0} + B_0Y_{x_0}}$ to be minimized.

Theorem 2: Suppose Assumption 1-3 are met and we determine $\boldsymbol{K_0}$ from Theorem 1. For the leader's system $(\boldsymbol{6})$ if there exit $\boldsymbol{P_{x_0} = P_{x_0}^T > 0}$, and $\boldsymbol{P_{\eta_0} = P_{\eta_0}^T > 0}$, a matrix $\boldsymbol{Y_{\eta_0}}$ and positive constants $\boldsymbol{\alpha_{\eta_0}}$ ‘$\boldsymbol{\beta_{\eta_0}}$ ‘$\boldsymbol{\gamma_{\eta_0}}$ ‘$\boldsymbol{\varepsilon_{\eta_0}}$ ‘ $\boldsymbol{\omega_{\eta_0}}$ ‘$\boldsymbol{\alpha_{x_0}}$ ‘$\boldsymbol{\beta_{x_0}}$ ‘$\boldsymbol{\gamma_{x_0}}$ and $\boldsymbol{\omega_{x_0}}$ satisfying the following condition:

$$\Pi_{\eta_0} = \begin{bmatrix} \Pi_{\eta_{0}11} & \Pi_{\eta_{0}12} & \wp_{\eta_{0}1} & 0 \\ * & \Pi_{\eta_{0}22} & 0 & \wp_{\eta_{0}2} \\ * & * & q_{\eta_{0}1} & 0 \\ * & * & * & q_{\eta_{0}2} \end{bmatrix} < 0 \tag{38}$$

where

$$\Pi_{\eta_{0}11} = A_0^TP_{\eta_0} + P_{\eta_0}A_0 + C_0^TY_{\eta_0}^T + Y_{\eta_0}C_0 + (\alpha_{\eta_0} + \gamma_{x_0})K_0^T\tilde{B}_0^T\tilde{B}_0K_0 + \omega_{\eta_0}\theta_0^2I_n + G_{\eta_0} + K_0^TG_{u_0}K_0$$

$$\Pi_{\eta_{0}12} = -K_0^TB_0^TP_{x_0} - K_0^TG_{u_0}K_0$$

$$\Pi_{\eta_{0}22} = (A_0 + B_0K_0)^TP_{x_0} + P_{x_0}(A_0 + B_0K_0) + (\alpha_{x_0} + \beta_{\eta_0})\tilde{A}_0^T\tilde{A}_0 + (\beta_{x_0} + \gamma_{\eta_0})K_0^T\tilde{B}_0^T\tilde{B}_0K_0 + \varepsilon_{\eta_0}\tilde{C}_0^T\tilde{C}_0 + \omega_{x_0}\theta_0^2I_n + K_0^TG_{u_0}K_0$$

$$\wp_{\eta_{0}1} = [P_{\eta_0}M_0 \quad P_{\eta_0}M_0 \quad P_{\eta_0}M_0 \quad Y_{\eta_0}N_0 \quad P_{\eta_0}]$$

$$\wp_{\eta_{0}2} = [P_{x_0}M_0 \quad P_{x_0}M_0 \quad P_{x_0}M_0 \quad P_{x_0}]$$

$q_{\eta_{0_1}} = -blockdiag(\alpha_{\eta_0}I_n, \beta_{\eta_0}I_n, \gamma_{\eta_0}I_n, \varepsilon_{\eta_0}I_n, \omega_{\eta_0}I_n)$
$q_{\eta_{0_2}} = -blockdiag(\alpha_{x_0}I_n, \beta_{x_0}I_n, \gamma_{x_0}I_n, \omega_{x_0}I_n)$

Then, under control input (12) and observer (10) with $E_0 = P_{\eta_0}^{-1}Y_{\eta_0}$, leader's system and observer error are asymptotically stable with guaranteed cost upper bound $J_{x_0}^* = \eta_0^T(0)P_{\eta_0}\eta_0(0) + x_0^T(0)P_{x_0}x_0(0)$ for cost function $J_{x_0} = \lim_{t\to\infty}\left[I^\alpha\left(j_{\eta_o}(t) + j_{u_o}(t)\right)\right]$.

Proof: According to observer error definition $\eta_0(t) = x_0(t) - \hat{x}_0(t)$ we can get:

$$D^\alpha\eta_0(t) = (A_0 + E_0C_0 - \Delta B_0(t)K_0)\eta_0(t) + \left(\Delta A_0(t) + \Delta B_0(t)K_0 + E_0\Delta C_0(t)\right)x_0(t) + f_0(x_0(t)) - f_0(\hat{x}_0(t)) \tag{39}$$

Consider the following Lyapunov candidate function:

$$V_{\eta_0}(t) = \eta_0^T(t)P_{\eta_0}\eta_0(t) + x_0^T(t)P_{x_0}x_0(t) \tag{40}$$

where P_{η_0} and $P_{x_0}$are unknown symmetric positive-definite matrices.

Taking the α-order derivative and using Lemma 3 yields:

$$D^\alpha V_{\eta_0}(t) \le \Big((A_0 + E_0C_0 - \Delta B_0(t)K_0)\eta_0(t) + \left(\Delta A_0(t) + \Delta B_0(t)K_0 + E_0\Delta C_0(t)\right)x_0(t) + f_0(x_0(t)) - f_0(\hat{x}_0(t))\Big)^T P_{\eta_0}\eta_0(t) + \eta_0^T(t)P_{\eta_0}\Big((A_0 + E_0C_0 - \Delta B_0(t)K_0)\eta_0(t) + \left(\Delta A_0(t) + \Delta B_0(t)K_0 + E_0\Delta C_0(t)\right)x_0(t) + f_0(x_0(t)) - f_0(\hat{x}_0(t))\Big) + \Big(\left(A_0 + \Delta A_0(t) + (B_0 + \Delta B_0(t))K_0\right)x_0(t) - (B_0 + \Delta B_0(t))K_0\eta_0(t) + f_0(x_0(t))\Big)^T P_{x_0}x_0(t) + x_0^T(t)P_{x_0}\Big(\left(A_0 + \Delta A_0(t) + (B_0 + \Delta B_0(t))K_0\right)x_0(t) - (B_0 + \Delta B_0(t))K_0\eta_0(t) + f_0(x_0(t))\Big) \tag{41}$$

By using Lemma 4 yields:

$$D^\alpha V_{\eta_0}(t) \le \begin{bmatrix}\eta_0(t)\\ x_0(t)\end{bmatrix}^T \Sigma_{\eta_0}\begin{bmatrix}\eta_0(t)\\ x_0(t)\end{bmatrix} \tag{42}$$

where

$$\Sigma_{\eta_0} = \begin{bmatrix}\Sigma_{\eta_{0_{11}}} & \Sigma_{\eta_{0_{12}}}\\ * & \Sigma_{\eta_{0_{22}}}\end{bmatrix}$$

$$\Sigma_{\eta_{0_{11}}} = (A_0 + E_0C_0)^TP_{\eta_0} + P_{\eta_0}(A_0 + E_0C_0) + (\alpha_{\eta_0} + \gamma_{x_0})K_0^T\tilde{B}_0^T\tilde{B}_0K_0 + \omega_{\eta_0}\theta_0^2I_n + \omega_{\eta_0}^{-1}P_{\eta_0}P_{\eta_0} + (\alpha_{\eta_0}^{-1} + \beta_{\eta_0}^{-1} + \gamma_{\eta_0}^{-1})P_{\eta_0}M_0M_0^TP_{\eta_0} + \varepsilon_{\eta_0}^{-1}P_{\eta_0}E_0N_0N_0^TE_0^TP_{\eta_0}$$
$$\Sigma_{\eta_{0_{12}}} = -K_0^TB_0^TP_{x_0}$$

$$\Sigma_{\eta_{0_{22}}} = (A_0 + B_0K_0)^TP_{x_0} + P_{x_0}(A_0 + B_0K_0) + (\alpha_{x_0} + \beta_{\eta_0})\tilde{A}_0^T\tilde{A}_0 + (\beta_{x_0} + \gamma_{\eta_0})K_0^T\tilde{B}_0^T\tilde{B}_0K_0 + (\alpha_{x_0}^{-1} + \beta_{x_0}^{-1} + \gamma_{x_0}^{-1})P_{x_0}M_0M_0^TP_{x_0} + \varepsilon_{\eta_0}\tilde{C}_0^T\tilde{C}_0 + \omega_{x_0}\theta_0^2I_n + \omega_{x_0}^{-1}P_{x_0}P_{x_0}$$

According to the fractional Lyapunov direct method, the leader's observer error is asymptotically stable If $\Sigma_{\eta_0} < 0$.

Furthermore, we consider the following cost function:

$$J_{x_0} = \lim_{t\to\infty}\left[I^\alpha\left(j_{\eta_o}(t) + j_{u_o}(t)\right)\right] \tag{43}$$

$$j_{\eta_0}(t) = \eta_0^T(t)G_{\eta_0}\eta_0(t) \tag{44}$$

$$j_{u_0}(t) = u_0^T(t)G_{u_0}u_0(t) = \left(x_0(t) - \eta_0(t)\right)^T K_0^TG_{u_0}K_0\left(x_0(t) - \eta_0(t)\right) \tag{45}$$

If $D^\alpha V_{\eta_0}(t) + j_{\eta_0}(t) + j_{u_0}(t) < 0$:

$$j_{\eta_o}(t) + j_{u_0}(t) < -D^\alpha V_{\eta_0}(t) \tag{46}$$

Then for $t \in [0,\infty)$, the α-order integrating both sides yields:

$$I^\alpha\left(j_{\eta_o}(t) + j_{u_o}(t)\right) < V_{\eta_0}(0) - V_{\eta_0}(t) \tag{47}$$

Since $D^\alpha V_{\eta_0}(t) < 0$, $\lim_{t\to\infty} V_{\eta_0}(t) = 0$, therefore:

$$J_{x_0} = \lim_{t\to\infty}\left[I^\alpha\left(j_{\eta_o}(t) + j_{u_o}(t)\right)\right] < V_{\eta_0}(0) = \eta_0^T(0)P_{\eta_0}\eta_0(0) + x_0^T(0)P_{x_0}x_0(0) = J_{x_0}^* \tag{48}$$

So, the upper bound $J_{x_0}^*$ of the quadratic guaranteed cost function can be obtained.

$$D^\alpha V_{\eta_0}(t) + j_{\eta_o}(t) + j_{u_0}(t) \le \begin{bmatrix}\eta_0(t)\\ x_0(t)\end{bmatrix}^T \hat{\Sigma}_{\eta_0}\begin{bmatrix}\eta_0(t)\\ x_0(t)\end{bmatrix} \tag{49}$$

where

$$\hat{\Sigma}_{\eta_0} = \begin{bmatrix}\hat{\Sigma}_{\eta_{0_{11}}} & \hat{\Sigma}_{\eta_{0_{12}}}\\ * & \hat{\Sigma}_{\eta_{0_{22}}}\end{bmatrix}$$

$$\hat{\Sigma}_{\eta_{0_{11}}} = (A_0 + E_0C_0)^TP_{\eta_0} + P_{\eta_0}(A_0 + E_0C_0) + (\alpha_{\eta_0} + \gamma_{x_0})K_0^T\tilde{B}_0^T\tilde{B}_0K_0 + \omega_{\eta_0}\theta_0^2I_n + \omega_{\eta_0}^{-1}P_{\eta_0}P_{\eta_0} + (\alpha_{\eta_0}^{-1} + \beta_{\eta_0}^{-1} + \gamma_{\eta_0}^{-1})P_{\eta_0}M_0M_0^TP_{\eta_0} + \varepsilon_{\eta_0}^{-1}P_{\eta_0}E_0N_0N_0^TE_0^TP_{\eta_0} + G_{\eta_0} + K_0^TG_{u_0}K_0$$
$$\hat{\Sigma}_{\eta_{0_{12}}} = -K_0^TB_0^TP_{x_0} - K_0^TG_{u_0}K_0$$

$$\hat{\Sigma}_{\eta_{0_{22}}} = (A_0 + B_0K_0)^TP_{x_0} + P_{x_0}(A_0 + B_0K_0) + (\alpha_{x_0} + \beta_{\eta_0})\tilde{A}_0^T\tilde{A}_0 + (\beta_{x_0} + \gamma_{\eta_0})K_0^T\tilde{B}_0^T\tilde{B}_0K_0 + (\alpha_{x_0}^{-1} + \beta_{x_0}^{-1} + \gamma_{x_0}^{-1})P_{x_0}M_0M_0^TP_{x_0} + \varepsilon_{\eta_0}\tilde{C}_0^T\tilde{C}_0 + \omega_{x_0}\theta_0^2I_n + \omega_{x_0}^{-1}P_{x_0}P_{x_0} + K_0^TG_{u_0}K_0$$

By employing Lemma 6 and let $\boldsymbol{Y_{\eta_0}} = \boldsymbol{P_{\eta_0}E_0}$, we can obtain (38) and this completes the proof.

Step 2: Stability of virtual consensus errors

In this step, we determine F_i and T_i considering to stability of virtual consensus errors.

Theorem 3: Suppose Assumption 1-4 are met and we determine $\boldsymbol{K_0}$ and $\boldsymbol{E_0}$ from Step 1. For virtual systems (11) if there exist matrices $\boldsymbol{X_\zeta} = \boldsymbol{X_\zeta^T} > \boldsymbol{0}$, and $\boldsymbol{\hat{T}_i} = \boldsymbol{\hat{T}_i}^{\boldsymbol{T}} > \boldsymbol{0}$, a matrix $\boldsymbol{Y_\zeta}$ and positive constants $\boldsymbol{\omega_{\zeta_i}}$ satisfying the following condition:

$$\Pi_\zeta = \begin{bmatrix}\Pi_{\zeta_{11}} & Y_\zeta & p_{\zeta_1}\\ * & -\hat{T} & p_{\zeta_2}\\ * & * & q_{\zeta_1}\end{bmatrix} < 0 \tag{50}$$

where

$$\Pi_{\zeta_{11}} = I_N\otimes\left(X_\zeta(A_0 + B_0K_0)^T + (A_0 + B_0K_0)X_\zeta\right) + \omega_\zeta + Y_\zeta^T + Y_\zeta$$

$$p_{\zeta_1} = \begin{bmatrix}I_N\otimes X_\zeta & (I_N\otimes X_\zeta)(H^T\otimes I_n) & I_N\otimes X_\zeta & Y_\zeta^T\end{bmatrix}$$

$$\mathcal{p}_{\zeta_2} = \begin{bmatrix} 0 & (I_N \otimes X_\zeta)(H^T \otimes I_n) & 0 & Y_\zeta^T \end{bmatrix}$$

$$\mathcal{q}_{\zeta_1} = -blockdiag\left(\omega_\zeta \theta_0^{-2}, \rho^{-1} Q^{-1}, G_\zeta^{-1}, G_v^{-1}\right)$$

Then, under the event-triggered consensus control protocol (14) with $F_i = Y_i X_\zeta^{-1}$ and $T_i = X_\zeta^{-1} \hat{T}_i X_\zeta^{-1}$, virtual consensus errors are asymptotically stable with guaranteed cost upper bound $J_{\check{x}}^* = \zeta^T(0)\left(I_N \otimes P_\zeta\right)\zeta(0)$ for cost function $J_{\check{x}} = \lim_{t\to\infty}\left[I^\alpha\left(j_\zeta(t) + j_v(t)\right)\right]$.

Proof: According virtual consensus errors definition $\zeta_i(t) = \check{x}_i(t) - \hat{x}_0(t)$ we can get:

$$D^\alpha \zeta_i(t) = (A_0 + B_0 K_0)\zeta_i(t) + F_i v_i(t) + f_0(\check{x}_i(t)) - f_0(\hat{x}_0(t)) + E_0 C_0 \eta_0(t) + E_0 \Delta C_0(t) x_0(t) \tag{51}$$

Consider the following Lyapunov candidate function:

$$V_\zeta(t) = \sum_{i=1}^{N} V_{\zeta_i}(t), \qquad V_{\zeta_i}(t) = \zeta_i^T(t) P_\zeta \zeta_i(t) \tag{52}$$

where P_ζ is an unknown symmetric positive-definite matrix.

Taking the α-order derivative and using Lemma 3 yields:

$$D^\alpha V_{\zeta_i}(t) \le D^\alpha \zeta_i^T(t) P_\zeta \zeta_i(t) + \zeta_i^T(t) P_\zeta D^\alpha \zeta_i(t) = \left[(A_0 + B_0 K_0)\zeta_i(t) + F_i v_i(t) + f_0(\check{x}_i(t)) - f_0(\hat{x}_0(t)) + E_0 C_0 \eta_0(t) + E_0 \Delta C_0(t) x_0(t)\right]^T P_\zeta \zeta_i(t) + \zeta_i^T(t) P_\zeta \left[(A_0 + B_0 K_0)\zeta_i(t) + F_i v_i(t) + f_0(\check{x}_i(t)) - f_0(\hat{x}_0(t)) + E_0 C_0 \eta_0(t) + E_0 \Delta C_0(t) x_0(t)\right] \tag{53}$$

In Step 1 we prove $\eta_0(t)$ and $x_0(t)$ are asymptotically stable then:

$$\lim_{t\to\infty}\left(\eta_0^T(t) C_0^T E_0^T P_\zeta \zeta_i(t) + \zeta_i^T(t) P_\zeta E_0 C_0 \eta_0(t)\right) = 0 \tag{54}$$

$$\lim_{t\to\infty}\left(x_0^T(t) \Delta C_0^T(t) E_0^T P_\zeta \zeta_i(t) + \zeta_i^T(t) P_\zeta E_0 \Delta C_0(t) x_0(t)\right) = 0 \tag{55}$$

And we ignore these parts. Using Lemma 4, yields:

$$D^\alpha V_{\zeta_i}(t) \le \zeta_i^T(t)\left((A_0 + B_0 K_0)^T P_\zeta + P_\zeta(A_0 + B_0 K_0) + \omega_{\zeta_i}^{-1}\theta_0^2 I_n + \omega_{\zeta_i} P_\zeta P_\zeta\right)\zeta_i(t) + v_i^T(t) F_i^T P_\zeta \zeta_i(t) + \zeta_i^T(t) P_\zeta F_i v_i(t) \tag{56}$$

We can obtain the following from the event-triggered consensus control protocol:

$$v_i(t) = \sum_{j=1}^{N} a_{ij}\left(\check{x}_i(t_{k_i}^i) - \check{x}_j\left(t_{k_j}^j\right)\right) + b_i\left(\check{x}_i(t_{k_i}^i) - \hat{x}_0(t)\right) = \sum_{j=1}^{N} a_{ij}\left(\check{x}_i(t) + e_i(t) - \check{x}_j(t) - e_j(t)\right) + b_i\left(\check{x}_i(t) + e_i(t) - \hat{x}_0(t)\right) = \sum_{j=1}^{N} a_{ij}\left(\zeta_i(t) + e_i(t) - \zeta_j(t) - e_j(t)\right) + b_i\left(\zeta_i(t) + e_i(t)\right) = \sum_{j=1}^{N} h_{ij}\left(e_j(t) + \zeta_j(t)\right) \tag{57}$$

This yields that:

$$D^\alpha V_\zeta(t) \le \sum_{i=1}^{N}\Bigg(\zeta_i^T(t)\left((A_0 + B_0 K_0)^T P_\zeta + P_\zeta(A_0 + B_0 K_0) + \omega_{\zeta_i}^{-1}\theta_0^2 I_n + \omega_{\zeta_i} P_\zeta P_\zeta\right)\zeta_i(t) + \left[\sum_{j=1}^{N} h_{ij} F_i\left(e_j(t) + \zeta_j(t)\right)\right]^T P_\zeta \zeta_i(t) + \zeta_i^T(t) P_\zeta\left[\sum_{j=1}^{N} h_{ij} F_i\left(e_j(t) + \zeta_j(t)\right)\right]\Bigg) \tag{58}$$

where

$$\zeta(t) = [\zeta_1^T(t), \dots, \zeta_N^T(t)]^T \in \mathbb{R}^{Nn\times 1}$$

$$e(t) = [e_1^T(t), \dots, e_N^T(t)]^T \in \mathbb{R}^{Nn\times 1}$$

$$F = [(h_1 \otimes F_1)^T, \dots, (h_N \otimes F_N)^T]^T \in \mathbb{R}^{Nn\times Nn}, \quad h_i \triangleq ith\ row\ of\ \mathcal{H}$$

$$\omega_\zeta = blockdiag\left(\omega_{\zeta_1} I_n, \omega_{\zeta_2} I_n, \dots, \omega_{\zeta_N} I_n\right) \in \mathbb{R}^{Nn\times Nn}$$

$$T = blockdiag(T_1, T_2, \dots, T_N) \in \mathbb{R}^{Nn\times Nn}$$

$$Q = blockdiag(Q_1, Q_2, \dots, Q_N) \in \mathbb{R}^{Nn\times Nn}$$

$$G_\zeta = blockdiag(G_{\zeta_1}, \dots, G_{\zeta_N})$$

$$G_v = blockdiag(G_{v_1}, \dots, G_{v_N})$$

Then (58) can be written into a compact form as follows:

$$D^\alpha V_\zeta(t) \le \zeta^T(t)\left(I_N \otimes \left((A_0 + B_0 K_0)^T P_\zeta + P_\zeta(A_0 + B_0 K_0)\right) + (I_N \otimes P_\zeta)\omega_\zeta(I_N \otimes P_\zeta) + \omega_\zeta^{-1}\theta_0^2\right)\zeta(t) + \left(\zeta(t) + e(t)\right)^T F^T (I_N \otimes P_\zeta)\zeta(t) + \zeta^T(t)(I_N \otimes P_\zeta) F\left(\zeta(t) + e(t)\right) \tag{59}$$

Based on conditions (15) and (59) the following inequality is derived:

$$D^\alpha V_\zeta(t) \le \begin{bmatrix}\zeta(t)\\ e(t)\end{bmatrix}^T \Sigma_\zeta \begin{bmatrix}\zeta(t)\\ e(t)\end{bmatrix} + \sum_{i=1}^{N}\{e_i^T(t) T_i e_i(t) - \rho v_i^T(t) Q_i v_i(t)\} = \begin{bmatrix}\zeta(t)\\ e(t)\end{bmatrix}^T \Sigma_\zeta \begin{bmatrix}\zeta(t)\\ e(t)\end{bmatrix} + e^T(t) T e(t) - \rho\sum_{i=1}^{N}\left\{\left[\sum_{j=1}^{N} h_{ij}\left(\zeta_j(t) + e_j(t)\right)\right]^T Q_i \left[\sum_{j=1}^{N} h_{ij}\left(\zeta_j(t) + e_j(t)\right)\right]\right\} = \begin{bmatrix}\zeta(t)\\ e(t)\end{bmatrix}^T \Sigma_\zeta \begin{bmatrix}\zeta(t)\\ e(t)\end{bmatrix} + e^T(t) T e(t) - \rho\left(\zeta(t) + e(t)\right)^T (H^T \otimes I_q) Q (H \otimes I_q)\left(\zeta(t) + e(t)\right) \tag{60}$$

where

$$\Sigma_\zeta = \begin{bmatrix}\Sigma_{\zeta_{11}} & \Sigma_{\zeta_{12}}\\ * & \Sigma_{\zeta_{22}}\end{bmatrix}$$

$$\Sigma_{\zeta_{11}} = I_N \otimes \left((A_0 + B_0 K_0)^T P_\zeta + P_\zeta(A_0 + B_0 K_0)\right) + \omega_\zeta^{-1}\theta_0^2 + (I_N \otimes P_\zeta)\omega_\zeta(I_N \otimes P_\zeta) + F^T(I_N \otimes P_\zeta) + (I_N \otimes P_\zeta) F + \rho(H^T \otimes I_q) Q (H \otimes I_q)$$

$$\Sigma_{\zeta_{12}} = (I_N \otimes P_\zeta) F + \rho(H^T \otimes I_q) Q (H \otimes I_q)$$

$$\Sigma_{\zeta_{22}} = -T + \rho(H^T \otimes I_q) Q (H \otimes I_q)$$

According to the fractional Lyapunov direct method, virtual consensus errors are asymptotically stable if $\Sigma_\zeta < 0$.

Furthermore, we consider the following cost function:

$$J_{\check{x}} = \lim_{t\to\infty}\left[I^\alpha\left(j_\zeta(t) + j_v(t)\right)\right] \tag{61}$$

$$j_\zeta(t) = \sum_{i=1}^{N}\{\zeta_i^T(t) G_{\zeta_i} \zeta_i(t)\} = \zeta^T(t) G_\zeta \zeta(t) \tag{62}$$

$$j_v(t) = \sum_{i=1}^{N}\{v_i^T(t)F_i^T G_{v_i} F_i v_i(t)\} = \big(\zeta(t) + e(t)\big)^T F^T G_v F\big(\zeta(t) + e(t)\big) \quad (63)$$

If $D^\alpha V_\zeta(t) + j_\zeta(t) + j_v(t) < 0$:

$$j_\zeta(t) + j_v(t) < -D^\alpha V_\zeta(t) \quad (64)$$

Then for $t \in [0, \infty)$, the α-order integrating both sides yields:

$$I^\alpha\left(j_\zeta(t) + j_v(t)\right) < V_\zeta(0) - V_\zeta(t) \quad (65)$$

Since $D^\alpha V_\zeta(t) < 0$, $\lim_{t\to\infty} V_\zeta(t) = 0$, therefore:

$$J_{\check{x}} = \left[I^\alpha\left(j_\zeta(t) + j_v(t)\right)\right] < V_\zeta(0) = \zeta^T(0)\big(I_N \otimes P_\zeta\big)\zeta(0) = J_{\check{x}}^* \quad (66)$$

So, the upper bound $J_{\check{x}}^*$ of the quadratic guaranteed cost function can be obtained.

$$D^\alpha V_\zeta(t) + j_\zeta(t) + j_v(t) \le \begin{bmatrix}\zeta(t)\\ e(t)\end{bmatrix}^T \hat{\Sigma}_\zeta \begin{bmatrix}\zeta(t)\\ e(t)\end{bmatrix} + \sum_{i=1}^{N}\{e_i^T(t)T_i e_i(t) - \rho v_i^T(t)Q_i v_i(t)\} \quad (67)$$

where

$$\hat{\Sigma}_\zeta = \begin{bmatrix}\hat{\Sigma}_{\zeta_{11}} & \hat{\Sigma}_{\zeta_{12}}\\ * & \hat{\Sigma}_{\zeta_{22}}\end{bmatrix}$$

$$\hat{\Sigma}_{\zeta_{11}} = I_N \otimes \left((A_0 + B_0K_0)^T P_\zeta + P_\zeta(A_0 + B_0K_0)\right) + \omega_\zeta^{-1}\theta_0^2 + \big(I_N \otimes P_\zeta\big)\omega_\zeta\big(I_N \otimes P_\zeta\big) + F^T\big(I_N \otimes P_\zeta\big) + \big(I_N \otimes P_\zeta\big)F + \rho\big(H^T \otimes I_q\big)Q\big(H \otimes I_q\big) + G_\zeta + F^T G_v F$$

$$\hat{\Sigma}_{\zeta_{12}} = \big(I_N \otimes P_\zeta\big)F + \rho\big(H^T \otimes I_q\big)Q\big(H \otimes I_q\big) + F^T G_v F$$

$$\hat{\Sigma}_{\zeta_{22}} = -T + \rho\big(H^T \otimes I_q\big)Q\big(H \otimes I_q\big) + F^T G_v F$$

For linearization of the inequality $\hat{\Sigma}_\zeta < 0$ by using Lemma 5 and pre- and post-multiplying both sides of the inequality by $T_\zeta = I_{2\times N} \otimes P_\zeta^{-1}$ and let $X_\zeta = P_\zeta^{-1}$, $Y_i = F_i P_\zeta^{-1}$, $Y_\zeta = [(h_1 \otimes Y_1)^T, \dots, (h_N \otimes Y_N)^T]^T$, $\hat{T}_i = X_\zeta T_i X_\zeta$ and $\hat{T} = blockdiag\big(\hat{T}_1, \hat{T}_2, \dots, \hat{T}_N\big)$ the following obtain:

$$T_\zeta \hat{\Sigma}_\zeta T_\zeta = \check{\Sigma}_\zeta = \begin{bmatrix}\check{\Sigma}_{\zeta_{11}} & \check{\Sigma}_{\zeta_{12}}\\ * & \check{\Sigma}_{\zeta_{22}}\end{bmatrix} < 0 \quad (68)$$

where

$$\check{\Sigma}_{\zeta_{11}} = I_N \otimes \big(X_\zeta(A_0 + B_0K_0)^T + (A_0 + B_0K_0)X_\zeta\big) + \big(I_N \otimes X_\zeta\big)\omega_\zeta^{-1}\theta_0^2\big(I_N \otimes X_\zeta\big) + \omega_\zeta + Y_\zeta^T + Y_\zeta + \rho\big(I_N \otimes X_\zeta\big)\big(H^T \otimes I_q\big)Q\big(H \otimes I_q\big)\big(I_N \otimes X_\zeta\big) + \big(I_N \otimes X_\zeta\big)G_\zeta\big(I_N \otimes X_\zeta\big) + Y_\zeta^T G_v Y_\zeta$$

$$\check{\Sigma}_{\zeta_{12}} = Y_\zeta + \rho\big(I_N \otimes X_\zeta\big)\big(H^T \otimes I_q\big)Q\big(H \otimes I_q\big)\big(I_N \otimes X_\zeta\big) + Y_\zeta^T G_v Y_\zeta$$

$$\check{\Sigma}_{\zeta_{22}} = -\hat{T} + \rho\big(I_N \otimes X_\zeta\big)\big(H^T \otimes I_q\big)Q\big(H \otimes I_q\big)\big(I_N \otimes X_\zeta\big) + Y_\zeta^T G_v Y_\zeta$$

By employing Lemma 6, we can obtain (50) and this completes the proof.

Step 3: Stability of followers' systems and consensus errors and observer errors for followers.

In this step for each follower first, we determine K_i and W_i considering to stability of follower's system and consensus errors then we determine E_i considering to dynamics of followers' system and stability of observers' errors.

Theorem 4: Suppose Assumption 1-4 are met and we determine $\boldsymbol{K_0}$ and $\boldsymbol{E_0}$ from Step 1 and $\boldsymbol{F_i}$ and $\boldsymbol{T_i}$ from Step 2 and $\boldsymbol{\eta_i(t) = 0}$. For $\boldsymbol{i}$th follower's system ($\boldsymbol{6}$) if there exit matrices $\boldsymbol{X_{x_i} = X_{x_i}^T > 0}$, and $\boldsymbol{X_{\varphi_i} = X_{\varphi_i}^T > 0}$ and matrices $\boldsymbol{Y_{x_i}}$ and $\boldsymbol{Y_{\varphi_i}}$ and positive constants $\boldsymbol{\alpha_{\varphi_i}}$, $\boldsymbol{\beta_{\varphi_i}}$, $\boldsymbol{\gamma_{\varphi_i}}$, $\boldsymbol{\omega_{\varphi_i}}$, $\boldsymbol{\alpha_{x_i}}$, $\boldsymbol{\beta_{x_i}}$, $\boldsymbol{\gamma_{x_i}}$ and $\boldsymbol{\omega_{x_i}}$ satisfying the following condition:

$$\Pi_{\varphi_i} = \begin{bmatrix}\Pi_{\varphi_{i_{11}}} & \Pi_{\varphi_{i_{12}}} & \mathcal{P}_{\varphi_{i_1}} & 0 & Y_{\varphi_i}^T\\ * & \Pi_{\varphi_{i_{22}}} & 0 & \mathcal{P}_{\varphi_{i_2}} & Y_{x_i}^T\\ * & * & \mathcal{q}_{\varphi_{i_1}} & 0 & 0\\ * & * & * & \mathcal{q}_{\varphi_{i_2}} & 0\\ * & * & * & * & -G_{u_i}^{-1}\end{bmatrix} < 0 \quad (69)$$

where

$$\Pi_{\varphi_{i_{11}}} = X_{\varphi_i}(A_0 + B_0K_0)^T + (A_0 + B_0K_0)X_{\varphi_i} + Y_{\varphi_i}^T B_i^T + B_i Y_{\varphi_i} + (\alpha_{\varphi_i} + \beta_{\varphi_i} + \gamma_{\varphi_i})M_i M_i^T + \omega_{\varphi_i} I_n$$

$$\Pi_{\varphi_{i_{12}}} = (A_i - A_0 - B_0K_0)X_{x_i} + B_i Y_{x_i} + Y_{\varphi_i}^T B_i^T$$

$$\Pi_{\varphi_{i_{22}}} = X_{x_i}A_i^T + A_i X_{x_i} + Y_{x_i}^T B_i^T + B_i Y_{x_i} + \big(\alpha_{x_i} + \beta_{x_i} + \gamma_{x_i}\big)M_i M_i^T + \omega_{x_i} I_{n_i}$$

$$\mathcal{P}_{\varphi_{i_1}} = \begin{bmatrix}Y_{\varphi_i}^T \tilde{B}_i^T & Y_{\varphi_i}^T \tilde{B}_i^T & X_{\varphi_i} & X_{\varphi_i}\end{bmatrix}$$

$$\mathcal{P}_{\varphi_{i_2}} = \begin{bmatrix}X_{x_i}\tilde{A}_i^T & X_{x_i}\tilde{A}_i^T & Y_{x_i}^T \tilde{B}_i^T & Y_{x_i}^T \tilde{B}_i^T & X_{x_i}\end{bmatrix}$$

$$\mathcal{q}_{\varphi_{i_1}} = -blockdiag\big(\alpha_{\varphi_i} I_n, \gamma_{x_i} I_n, \omega_{\varphi_i}\sigma_i^{-2} I_n, G_{\varphi_i}^{-1}\big)$$

$$\mathcal{q}_{\varphi_{i_2}} = -blockdiag\big(\alpha_{x_i} I_n, \beta_{\varphi_i} I_n, \beta_{x_i} I_n, \gamma_{\varphi_i} I_n, \omega_{x_i}\theta_i^{-2} I_n\big)$$

Then, under control input (12) with $K_i = Y_{x_i} X_{x_i}^{-1}$ and $W_i = Y_{\varphi_i} X_{\varphi_i}^{-1}$, ith follower's system and consensus error is asymptotically stable with guaranteed cost upper bound $J_{x_i}^* = \varphi_i^T(0)P_{\varphi_i}\varphi_i(0) + x_i^T(0)P_{x_i}x_i(0)$ for cost function $J_{x_i} = \lim_{t\to\infty}\left[I^\alpha\left(j_{\varphi_i}(t) + j_{u_i}(t)\right)\right]$.

Proof: Apply the control input (12) to ith follower's system (6) yields:

$$D^\alpha x_i(t) = (A_i + \Delta A_i + (B_i + \Delta B_i)K_i)x_i(t) - (B_i + \Delta B_i)K_i\eta_i(t) + (B_i + \Delta B_i)W_i\big(\varphi_i(t) - \eta_i(t)\big) + f_i\big(x_i(t)\big) \quad (70)$$

According to the ith follower's consensus errors definition $\varphi_i(t) = x_i(t) - \check{x}_i(t)$ we can get:

$$D^\alpha \varphi_i(t) = (A_0 + B_0K_0 + (B_i + \Delta B_i)W_i)\varphi_i(t) + (A_i + B_iK_i + \Delta A_i + \Delta B_iK_i - A_0 - B_0K_0)x_i(t) - (B_i + \Delta B_i)(K_i + W_i)\eta_i(t) + f_i\big(x_i(t)\big) - f_0\big(\check{x}_i(t)\big) - F_i v_i(t) \quad (71)$$

Consider the following Lyapunov candidate function:

$$V_{\varphi_i}(t) = \varphi_i^T(t)P_{\varphi_i}\varphi_i(t) + x_i^T(t)P_{x_i}x_i(t) \quad (72)$$

where P_{φ_i} and P_{x_i} are unknown symmetric positive-definite matrices.

Taking the α-order derivative and using Lemma 3 yields:

$$D^\alpha V_{\varphi_i}(t) \le \Big((A_0 + B_0K_0 + (B_i + \Delta B_i)W_i)\varphi_i(t) + (A_i + B_iK_i + \Delta A_i + \Delta B_iK_i - A_0 - B_0K_0)x_i(t) - (B_i + \Delta B_i)(K_i + W_i)\eta_i(t) + f_i\big(x_i(t)\big) - f_0\big(\check{x}_i(t)\big) - F_i v_i(t)\Big)^T P_{\varphi_i}\varphi_i(t) + \varphi_i^T(t)P_{\varphi_i}\Big((A_0 + B_0K_0 + (B_i + \Delta B_i)W_i)\varphi_i(t) + (A_i + B_iK_i + \Delta A_i + \Delta B_iK_i - A_0 - B_0K_0)x_i(t) - (B_i + \Delta B_i)(K_i + W_i)\eta_i(t) + f_i\big(x_i(t)\big) - f_0\big(\check{x}_i(t)\big) - \quad (73)$$

$F_i v_i(t)\Big) + \Big((A_i + \Delta A_i + (B_i + \Delta B_i)K_i)x_i(t) - (B_i + \Delta B_i)K_i\eta_i(t) + (B_i + \Delta B_i)W_i\big(\varphi_i(t) - \eta_i(t)\big) + f_i\big(x_i(t)\big)\Big)^T P_{x_i}x_i(t) + x_i^T(t)P_{x_i}\Big((A_i + \Delta A_i + (B_i + \Delta B_i)K_i)x_i(t) - (B_i + \Delta B_i)K_i\eta_i(t) + (B_i + \Delta B_i)W_i\big(\varphi_i(t) - \eta_i(t)\big) + f_i\big(x_i(t)\big)\Big)$

In Step 2 we prove $\zeta(t)$ is asymptotically stable so $v_i(t) = \sum_{j=1}^{N} h_{ij}\left(e_j(t) + \zeta_j(t)\right)$ is asymptotically stable then:

$$\lim_{t\to\infty}\left(-v_i^T(t)F_i^T P_{\varphi_i}\varphi_i(t) - \varphi_i^T(t)P_{\varphi_i}F_i v_i(t)\right) = 0 \qquad (74)$$

and we ignore this part. Assuming $\eta_i(t) = 0$ and using Lemma 4 yields:

$$D^\alpha V_{\varphi_i}(t) \le \begin{bmatrix}\varphi_i(t)\\ x_i(t)\end{bmatrix}^T \Sigma_{\varphi_i}\begin{bmatrix}\varphi_i(t)\\ x_i(t)\end{bmatrix} \qquad (75)$$

where

$$\Sigma_{\varphi_i} = \begin{bmatrix}\Sigma_{\varphi_{i_{11}}} & \Sigma_{\varphi_{i_{12}}}\\ * & \Sigma_{\varphi_{i_{22}}}\end{bmatrix}$$

$\Sigma_{\varphi_{i_{11}}} = (A_0 + B_0K_0 + B_iW_i)^T P_{\varphi_i} + P_{\varphi_i}(A_0 + B_0K_0 + B_iW_i) + \left(\alpha_{\varphi_i}^{-1} + \gamma_{x_i}^{-1}\right)W_i^T\tilde{B}_i^T\tilde{B}_iW_i + \left(\alpha_{\varphi_i} + \beta_{\varphi_i} + \gamma_{\varphi_i}\right)P_{\varphi_i}M_iM_i^TP_{\varphi_i} + \omega_{\varphi_i}P_{\varphi_i}P_{\varphi_i} + \omega_{\varphi_i}^{-1}\sigma_i^2 I_n$

$\Sigma_{\varphi_{i_{12}}} = P_{\varphi_i}(A_i + B_iK_i - A_0 - B_0K_0) + W_i^TB_i^TP_{x_i}$

$\Sigma_{\varphi_{i_{22}}} = (A_i + B_iK_i)^TP_{x_i} + P_{x_i}(A_i + B_iK_i) + \left(\alpha_{x_i}^{-1} + \beta_{\varphi_i}^{-1}\right)\tilde{A}_i^T\tilde{A}_i + \left(\beta_{x_i}^{-1} + \gamma_{\varphi_i}^{-1}\right)K_i^T\tilde{B}_i^T\tilde{B}_iK_i + \left(\alpha_{x_i} + \beta_{x_i} + \gamma_{x_i}\right)P_{x_i}M_iM_i^TP_{x_i} + \omega_{x_i}^{-1}\theta_i^2I_n + \omega_{x_i}P_{x_i}P_{x_i}$

According to the fractional Lyapunov direct method, ith follower's system and consensus error are asymptotically stable If $\Sigma_{\varphi_i} < 0$.

Furthermore, we consider the following cost function:

$$J_{x_i} = \lim_{t\to\infty}\left[I^\alpha\left(j_{\varphi_i}(t) + j_{u_i}(t)\right)\right] \qquad (76)$$

$$j_{\varphi_i}(t) = \varphi_i^T(t)G_{\varphi_i}\varphi_i(t) \qquad (77)$$

$$j_{u_i}(t) = u_i^T(t)G_{u_i}u_i(t) = \left(K_ix_i(t) + W_i\varphi_i(t)\right)^TG_{u_i}\left(K_ix_i(t) + W_i\varphi_i(t)\right) \qquad (78)$$

If $D^\alpha V_{\varphi_i}(t) + j_{\varphi_i}(t) + j_{u_i}(t) < 0$:

$$j_{\varphi_i}(t) + j_{u_i}(t) < -D^\alpha V_{\varphi_i}(t) \qquad (79)$$

Then for $t \in [0, \infty)$, the α-order integrating both sides yields:

$$I^\alpha\left(j_{\varphi_i}(t) + j_{u_i}(t)\right) < V_{\varphi_i}(0) - V_{\varphi_i}(t) \qquad (80)$$

Since $D^\alpha V_{\varphi_i}(t) < 0$, $\lim_{t\to\infty} V_{\varphi_i}(t) = 0$, therefore:

$$J_{x_i} = \lim_{t\to\infty}\left[I^\alpha\left(j_{\varphi_i}(t) + j_{u_i}(t)\right)\right] < V_{\varphi_i}(0) = \varphi_i^T(0)P_{\varphi_i}\varphi_i(0) + x_i^T(0)P_{x_i}x_i(0) = J_{x_i}^* \qquad (81)$$

So, the upper bound $J_{x_i}^*$ of the quadratic guaranteed cost function can be obtained.

$$D^\alpha V_{\varphi_i}(t) + j_{\varphi_i}(t) + j_{u_i}(t) \le \begin{bmatrix}\varphi_i(t)\\ x_i(t)\end{bmatrix}^T\hat{\Sigma}_{\varphi_i}\begin{bmatrix}\varphi_i(t)\\ x_i(t)\end{bmatrix} \qquad (82)$$

where

$$\hat{\Sigma}_{\varphi_i} = \begin{bmatrix}\hat{\Sigma}_{\varphi_{i_{11}}} & \hat{\Sigma}_{\varphi_{i_{12}}}\\ * & \hat{\Sigma}_{\varphi_{i_{22}}}\end{bmatrix}$$

$\hat{\Sigma}_{\varphi_{i_{11}}} = (A_0 + B_0K_0 + B_iW_i)^TP_{\varphi_i} + P_{\varphi_i}(A_0 + B_0K_0 + B_iW_i) + \left(\alpha_{\varphi_i}^{-1} + \gamma_{x_i}^{-1}\right)W_i^T\tilde{B}_i^T\tilde{B}_iW_i + \left(\alpha_{\varphi_i} + \beta_{\varphi_i} + \gamma_{\varphi_i}\right)P_{\varphi_i}M_iM_i^TP_{\varphi_i} + \omega_{\varphi_i}P_{\varphi_i}P_{\varphi_i} + \omega_{\varphi_i}^{-1}\sigma_i^2I_n + G_{\varphi_i} + W_i^TG_{u_i}W_i$

$\hat{\Sigma}_{\varphi_{i_{12}}} = P_{\varphi_i}(A_i + B_iK_i - A_0 - B_0K_0) + W_i^TB_i^TP_{x_i} + W_i^TG_{u_i}K_i$

$\hat{\Sigma}_{\varphi_{i_{22}}} = (A_i + B_iK_i)^TP_{x_i} + P_{x_i}(A_i + B_iK_i) + \left(\alpha_{x_i}^{-1} + \beta_{\varphi_i}^{-1}\right)\tilde{A}_i^T\tilde{A}_i + \left(\beta_{x_i}^{-1} + \gamma_{\varphi_i}^{-1}\right)K_i^T\tilde{B}_i^T\tilde{B}_iK_i + \left(\alpha_{x_i} + \beta_{x_i} + \gamma_{x_i}\right)P_{x_i}M_iM_i^TP_{x_i} + \omega_{x_i}^{-1}\theta_i^2I_n + \omega_{x_i}P_{x_i}P_{x_i} + K_i^TG_{u_i}K_i$

For linearization of the inequality $\hat{\Sigma}_{\varphi_i} < 0$ by using Lemma 5 and pre- and post-multiplying both sides of the inequality by $T_{\varphi_i} = blockdiag\left(P_{\varphi_i}^{-1}, P_{x_i}^{-1}\right)$ and let $X_{\varphi_i} = P_{\varphi_i}^{-1}$, $X_{x_i} = P_{x_i}^{-1}$, $Y_{\varphi_i} = W_iP_{\varphi_i}^{-1}$ and $Y_{x_i} = K_iP_{x_i}^{-1}$, the following obtain:

$$T_{\varphi_i}\hat{\Sigma}_{\varphi_i}T_{\varphi_i} = \check{\Sigma}_{\varphi_i} = \begin{bmatrix}\check{\Sigma}_{\varphi_{i_{11}}} & \check{\Sigma}_{\varphi_{i_{12}}}\\ * & \check{\Sigma}_{\varphi_{i_{22}}}\end{bmatrix} \qquad (83)$$

where

$\check{\Sigma}_{\varphi_{i_{11}}} = X_{\varphi_i}(A_0 + B_0K_0)^T + (A_0 + B_0K_0)X_{\varphi_i} + Y_{\varphi_i}^TB_i^T + B_iY_{\varphi_i} + \left(\alpha_{\varphi_i}^{-1} + \gamma_{x_i}^{-1}\right)Y_{\varphi_i}^T\tilde{B}_i^T\tilde{B}_iY_{\varphi_i} + \left(\alpha_{\varphi_i} + \beta_{\varphi_i} + \gamma_{\varphi_i}\right)M_iM_i^T + \omega_{\varphi_i}I_n + \omega_{\varphi_i}^{-1}\sigma_i^2X_{\varphi_i}X_{\varphi_i} + X_{\varphi_i}G_{\varphi_i}X_{\varphi_i} + Y_{\varphi_i}^TG_{u_i}Y_{\varphi_i}$

$\check{\Sigma}_{\varphi_{i_{12}}} = (A_i - A_0 - B_0K_0)X_{x_i} + B_iY_{x_i} + Y_{\varphi_i}^TB_i^T + Y_{\varphi_i}^TG_{u_i}Y_{x_i}$

$\check{\Sigma}_{\varphi_{i_{22}}} = X_{x_i}A_i^T + A_iX_{x_i} + Y_{x_i}^TB_i^T + B_iY_{x_i} + \left(\alpha_{x_i}^{-1} + \beta_{\varphi_i}^{-1}\right)X_{x_i}\tilde{A}_i^T\tilde{A}_iX_{x_i} + \left(\beta_{x_i}^{-1} + \gamma_{\varphi_i}^{-1}\right)Y_{x_i}^T\tilde{B}_i^T\tilde{B}_iY_{x_i} + \left(\alpha_{x_i} + \beta_{x_i} + \gamma_{x_i}\right)M_iM_i^T + \omega_{x_i}^{-1}\theta_i^2X_{x_i}X_{x_i} + \omega_{x_i}I_n + Y_{x_i}^TG_{u_i}Y_{x_i}$

By employing Lemma 6, we can obtain (69) and this completes the proof.

Theorem 5: Suppose Assumption 1-4 are met and we determine $\boldsymbol{K_0}$ and $\boldsymbol{E_0}$ from Step 1 and $\boldsymbol{F_i}$ and $\boldsymbol{T_i}$ from Step 2 and $\boldsymbol{K_i}$ and $\boldsymbol{W_i}$ from Theorem 5. For $\boldsymbol{i}$th follower's system ($\boldsymbol{6}$) if there exit mtrices $\boldsymbol{P_{x_0} = P_{x_0}^T > 0}$, $\boldsymbol{P_{\varphi_i} = P_{\varphi_i}^T > 0}$ and $\boldsymbol{P_{\eta_i} = P_{\eta_i}^T > 0}$ and a matrix $\boldsymbol{Y_{\eta_i}}$ and positive constants $\boldsymbol{\alpha_{\eta_i}}$, $\boldsymbol{\beta_{\eta_i}}$, $\boldsymbol{\gamma_{\eta_i}}$, $\boldsymbol{\omega_{\eta_i}}$, $\boldsymbol{\varepsilon_{\eta_i}}$, $\boldsymbol{\epsilon_{\eta_i}}$, $\boldsymbol{\alpha_{\varphi_i}}$, $\boldsymbol{\beta_{\varphi_i}}$, $\boldsymbol{\gamma_{\varphi_i}}$, $\boldsymbol{\varepsilon_{\varphi_i}}$, $\boldsymbol{\omega_{\varphi_i}}$, $\boldsymbol{\alpha_{x_i}}$, $\boldsymbol{\beta_{x_i}}$, $\boldsymbol{\gamma_{x_i}}$, $\boldsymbol{\varepsilon_{x_i}}$ and $\boldsymbol{\omega_{x_i}}$ satisfying the following condition:

$$\Pi_{\eta_i} = \begin{bmatrix}\Pi_{\eta_{i_{11}}} & \Pi_{\eta_{i_{12}}} & \Pi_{\eta_{i_{13}}} & \mathcal{p}_{\eta_{i_1}} & 0 & 0\\ * & \Pi_{\eta_{i_{22}}} & \Pi_{\eta_{i_{23}}} & 0 & \mathcal{p}_{\eta_{i_2}} & 0\\ * & * & \Pi_{\eta_{i_{33}}} & 0 & 0 & \mathcal{p}_{\eta_{i_3}}\\ * & * & * & \mathcal{q}_{\eta_{i_1}} & 0 & 0\\ * & * & * & * & \mathcal{q}_{\eta_{i_2}} & 0\\ * & * & * & * & * & \mathcal{q}_{\eta_{i_3}}\end{bmatrix} < 0 \qquad (84)$$

where

$$\Pi_{\eta_{i_{11}}} = A_i^T P_{\eta_i} + P_{\eta_i} A_i + C_i^T Y_{\eta_i}^T + Y_{\eta_i} C_i + \left(\alpha_{\eta_i} + \varepsilon_{\varphi_i} + \varepsilon_{x_i}\right)(W_i + K_i)^T \tilde{B}_i^T \tilde{B}_i (W_i + K_i) + \omega_{\eta_i}\theta_i^2 I_n + G_{\eta_i} + (W_i + K_i)^T G_{u_i}(W_i + K_i)$$

$$\Pi_{\eta_{i_{12}}} = -(W_i + K_i)^T B_i^T P_{\varphi_i} - (W_i + K_i)^T G_{u_i} W_i$$

$$\Pi_{\eta_{i_{13}}} = -(W_i + K_i)^T B_i^T P_{x_i} - (W_i + K_i)^T G_{u_i} K_i$$

$$\Pi_{\eta_{i_{22}}} = (A_0 + B_0 K_0 + B_i W_i)^T P_{\varphi_i} + P_{\varphi_i}(A_0 + B_0 K_0 + B_i W_i) + \left(\alpha_{\varphi_i} + \gamma_{x_i} + \epsilon_{\eta_i}\right) W_i^T \tilde{B}_i^T \tilde{B}_i W_i + \omega_{\varphi_i}\sigma_i^2 I_n + W_i^T G_{u_i} W_i$$

$$\Pi_{\eta_{i_{23}}} = P_{\varphi_i}(A_i + B_i K_i - A_0 - B_0 K_0) + W_i^T B_i^T P_{x_i} + W_i^T G_{u_i} K_i$$

$$\Pi_{\eta_{i_{33}}} = (A_i + B_i K_i)^T P_{x_i} + P_{x_i}(A_i + B_i K_i) + \left(\alpha_{x_i} + \beta_{\varphi_i} + \beta_{\eta_i}\right)\tilde{A}_i^T \tilde{A}_i + \left(\beta_{x_i} + \gamma_{\varphi_i} + \gamma_{\eta_i}\right) K_i^T \tilde{B}_i^T \tilde{B}_i K_i + \omega_{x_i}\theta_i^2 I_n + \varepsilon_{\eta_i}\tilde{C}_i^T \tilde{C}_i + K_i^T G_{u_i} K_i$$

$$\mathscr{p}_{\eta_{i_1}} = \begin{bmatrix} P_{\eta_i}M_i & P_{\eta_i}M_i & P_{\eta_i}M_i & P_{\eta_i}M_i & Y_{\eta_i}N_i & P_{\eta_i} \end{bmatrix}$$

$$\mathscr{p}_{\eta_{i_2}} = \begin{bmatrix} P_{\varphi_i}M_i & P_{\varphi_i}M_i & P_{\varphi_i}M_i & P_{\varphi_i}M_i & P_{\varphi_i} \end{bmatrix}$$

$$\mathscr{p}_{\eta_{i_3}} = \begin{bmatrix} P_{x_i}M_i & P_{x_i}M_i & P_{x_i}M_i & P_{x_i}M_i & P_{x_i} \end{bmatrix}$$

$$\mathscr{q}_{\eta_{i_1}} = -blockdiag\left(\alpha_{\eta_i}I_n, \beta_{\eta_i}I_n, \gamma_{\eta_i}I_n, \epsilon_{\eta_i}I_n, \varepsilon_{\eta_i}I_n, \omega_{\eta_i}I_n\right)$$

$$\mathscr{q}_{\eta_{i_2}} = -blockdiag\left(\alpha_{\varphi_i}I_n, \beta_{\varphi_i}I_n, \gamma_{\varphi_i}I_n, \varepsilon_{\varphi_i}I_n, \omega_{\varphi_i}I_n\right)$$

$$\mathscr{q}_{\eta_{i_3}} = -blockdiag\left(\alpha_{x_i}I_n, \beta_{x_i}I_n, \gamma_{x_i}I_n, \varepsilon_{x_i}I_n, \omega_{x_i}I_n\right)$$

Then, under control input (12) and observer (10) with $E_i = P_{\eta_i}^{-1} Y_{\eta_i}$, ith follower's system and consensus error and observer error are asymptotically stable with guaranteed cost upper bound $J_{x_i}^* = \eta_i^T(0) P_{\eta_i}\eta_i(0) + \varphi_i^T(0) P_{\varphi_i}\varphi_i(0) + x_i^T(0) P_{x_i} x_i(0)$ for cost function $J_{x_i} = \lim_{t\to\infty}\left[I^\alpha\left(j_{\eta_i}(t) + j_{\varphi_i}(t) + j_{u_i}(t)\right)\right]$.

Proof: According to ith observer error definition $\eta_i(t) = x_i(t) - \hat{x}_i(t)$ we can get:

$$D^\alpha \eta_i(t) = \left(A_i + E_i C_i - \Delta B_i(K_i + W_i)\right)\eta_i(t) + (\Delta A_i + \Delta B_i K_i + E_i \Delta C_i) x_i(t) + \Delta B_i W_i \varphi_i(t) + f_i\left(x_i(t)\right) - f_i\left(\hat{x}_i(t)\right) \tag{85}$$

Consider the following Lyapunov candidate function:

$$V_{\eta_i}(t) = \eta_i^T(t) P_{\eta_i}\eta_i(t) + \varphi_i^T(t) P_{\varphi_i}\varphi_i(t) + x_i^T(t) P_{x_i} x_i(t) \tag{86}$$

where P_{η_i}, P_{φ_i} and $P_{x_i}$are unknown symmetric positive-definite matrices.

Taking the α-order derivative and using Lemma 3 yields:

$$\begin{aligned} D^\alpha V_{\eta_i}(t) \le{} & \Big(\left(A_i + E_i C_i - \Delta B_i(K_i + W_i)\right)\eta_i(t) + (\Delta A_i + \Delta B_i K_i + E_i \Delta C_i) x_i(t) + \Delta B_i W_i \varphi_i(t) + f_i\left(x_i(t)\right) - f_i\left(\hat{x}_i(t)\right)\Big)^T P_{\eta_i}\eta_i(t) \\ & + \eta_i^T(t) P_{\eta_i}\Big(\left(A_i + E_i C_i - \Delta B_i(K_i + W_i)\right)\eta_i(t) + (\Delta A_i + \Delta B_i K_i + E_i \Delta C_i) x_i(t) + \Delta B_i W_i \varphi_i(t) + f_i\left(x_i(t)\right) - f_i\left(\hat{x}_i(t)\right)\Big) \\ & + \Big(\left(A_0 + B_0 K_0 + (B_i + \Delta B_i) W_i\right)\varphi_i(t) + (A_i + B_i K_i + \Delta A_i + \Delta B_i K_i - A_0 - B_0 K_0) x_i(t) - (B_i + \Delta B_i)(K_i + W_i)\eta_i(t) + f_i\left(x_i(t)\right) - f_0\left(\check{x}_i(t)\right) - F_i v_i(t)\Big)^T P_{\varphi_i}\varphi_i(t) \\ & + \varphi_i^T(t) P_{\varphi_i}\Big(\left(A_0 + B_0 K_0 + (B_i + \Delta B_i) W_i\right)\varphi_i(t) + (A_i + B_i K_i + \Delta A_i + \Delta B_i K_i - A_0 - B_0 K_0) x_i(t) - (B_i + \Delta B_i)(K_i + W_i)\eta_i(t) + f_i\left(x_i(t)\right) - f_0\left(\check{x}_i(t)\right) - F_i v_i(t)\Big) \\ & + \Big(\left(A_i + \Delta A_i + (B_i + \Delta B_i) K_i\right) x_i(t) - (B_i + \Delta B_i) K_i \eta_i(t) + (B_i + \Delta B_i) W_i\left(\varphi_i(t) - \eta_i(t)\right) + f_i\left(x_i(t)\right)\Big)^T P_{x_i} x_i(t) \\ & + x_i^T(t) P_{x_i}\Big(\left(A_i + \Delta A_i + (B_i + \Delta B_i) K_i\right) x_i(t) - (B_i + \Delta B_i) K_i \eta_i(t) + (B_i + \Delta B_i) W_i\left(\varphi_i(t) - \eta_i(t)\right) + f_i\left(x_i(t)\right)\Big) \end{aligned} \tag{87}$$

By using Lemma 4 yields:

$$D^\alpha V_{\eta_i}(t) \le \begin{bmatrix} \eta_i(t) \\ \varphi_i(t) \\ x_i(t) \end{bmatrix}^T \Sigma_{\eta_i} \begin{bmatrix} \eta_i(t) \\ \varphi_i(t) \\ x_i(t) \end{bmatrix} \tag{88}$$

where

$$\Sigma_{\eta_i} = \begin{bmatrix} \Sigma_{\eta_{i_{11}}} & \Sigma_{\eta_{i_{12}}} & \Sigma_{\eta_{i_{13}}} \\ * & \Sigma_{\eta_{i_{22}}} & \Sigma_{\eta_{i_{23}}} \\ * & * & \Sigma_{\eta_{i_{33}}} \end{bmatrix}$$

$$\Sigma_{\eta_{i_{11}}} = (A_i + E_i C_i)^T P_{\eta_i} + P_{\eta_i}(A_i + E_i C_i) + \left(\alpha_{\eta_i} + \varepsilon_{\varphi_i} + \varepsilon_{x_i}\right)(W_i + K_i)^T \tilde{B}_i^T \tilde{B}_i (W_i + K_i) + \left(\alpha_{\eta_i}^{-1} + \beta_{\eta_i}^{-1} + \gamma_{\eta_i}^{-1} + \epsilon_{\eta_i}^{-1}\right) P_{\eta_i} M_i M_i^T P_{\eta_i} + \varepsilon_{\eta_i}^{-1} P_{\eta_i} E_i N_i N_i^T E_i^T P_{\eta_i} + \omega_{\eta_i}\theta_i^2 I_n + \omega_{\eta_i}^{-1} P_{\eta_i} P_{\eta_i}$$

$$\Sigma_{\eta_{i_{12}}} = -(W_i + K_i)^T B_i^T P_{\varphi_i}$$

$$\Sigma_{\eta_{i_{13}}} = -(W_i + K_i)^T B_i^T P_{x_i}$$

$$\Sigma_{\eta_{i_{22}}} = (A_0 + B_0 K_0 + B_i W_i)^T P_{\varphi_i} + P_{\varphi_i}(A_0 + B_0 K_0 + B_i W_i) + \left(\alpha_{\varphi_i} + \gamma_{x_i} + \epsilon_{\eta_i}\right) W_i^T \tilde{B}_i^T \tilde{B}_i W_i + \left(\alpha_{\varphi_i}^{-1} + \beta_{\varphi_i}^{-1} + \gamma_{\varphi_i}^{-1} + \varepsilon_{\varphi_i}^{-1}\right) P_{\varphi_i} M_i M_i^T P_{\varphi_i} + \omega_{\varphi_i}^{-1} P_{\varphi_i} P_{\varphi_i} + \omega_{\varphi_i}\sigma_i^2 I_n$$

$$\Sigma_{\eta_{i_{23}}} = P_{\varphi_i}(A_i + B_i K_i - A_0 - B_0 K_0) + W_i^T B_i^T P_{x_i}$$

$$\Sigma_{\eta_{i_{33}}} = (A_i + B_i K_i)^T P_{x_i} + P_{x_i}(A_i + B_i K_i) + \left(\alpha_{x_i} + \beta_{\varphi_i} + \beta_{\eta_i}\right)\tilde{A}_i^T \tilde{A}_i + \left(\beta_{x_i} + \gamma_{\varphi_i} + \gamma_{\eta_i}\right) K_i^T \tilde{B}_i^T \tilde{B}_i K_i + \left(\alpha_{x_i}^{-1} + \beta_{x_i}^{-1} + \gamma_{x_i}^{-1} + \varepsilon_{x_i}^{-1}\right) P_{x_i} M_i M_i^T P_{x_i} + \omega_{x_i}\theta_i^2 I_n + \omega_{x_i}^{-1} P_{x_i} P_{x_i} + \varepsilon_{\eta_i}\tilde{C}_i^T \tilde{C}_i$$

According to the fractional Lyapunov direct method, ith follower's observer error is asymptotically stable If $\Sigma_{\eta_i} < 0$.

Furthermore, we consider the following cost function:

$$J_{x_i} = \lim_{t\to\infty}\left[I^\alpha\left(j_{\eta_i}(t) + j_{\varphi_i}(t) + j_{u_i}(t)\right)\right] \tag{89}$$

$$j_{\eta_i}(t) = \eta_i^T(t) G_{\eta_i}\eta_i(t) \tag{90}$$

$$j_{\varphi_i}(t) = \varphi_i^T(t) G_{\varphi_i}\varphi_i(t) \tag{91}$$

$$j_{u_i}(t) = u_i^T(t) G_{u_i} u_i(t) = \Big(K_i\left(x_i(t) - \eta_i(t)\right) + W_i\left(\varphi_i(t) - \eta_i(t)\right)\Big)^T G_{u_i}\Big(K_i\left(x_i(t) - \eta_i(t)\right) + W_i\left(\varphi_i(t) - \eta_i(t)\right)\Big) \tag{92}$$

If $D^\alpha V_{\eta_i}(t) + j_{\eta_i}(t) + j_{\varphi_i}(t) + j_{u_i}(t) < 0$:

$$j_{\eta_i}(t) + j_{\varphi_i}(t) + j_{u_i}(t) < -D^\alpha V_{\eta_i}(t) \tag{93}$$

Then for $t \in [0, \infty)$, the α-order integrating both sides yields:

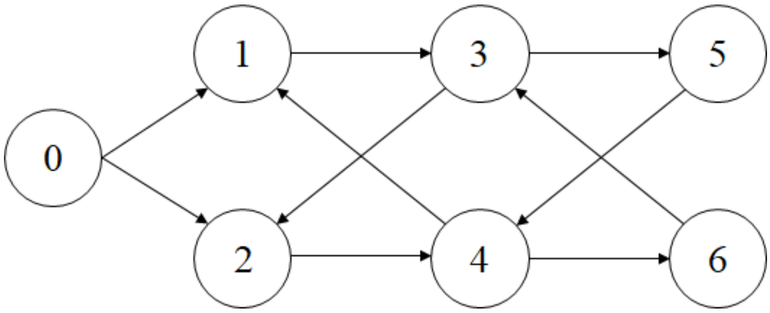

Figure 2: The directed interaction topology graph of the MAS.

$$I^{\alpha}\left(j_{\eta_i}(t)+j_{\varphi_i}(t)+j_{u_i}(t)\right)<V_{\eta_i}(0)-V_{\eta_i}(t) \quad (94)$$

Since $D^{\alpha}V_{\eta_i}(t)<0$, $\lim\limits_{t\to\infty}V_{\eta_i}(t)=0$, therefore:

$$J_{x_i}=\lim_{t\to\infty}\left[I^{\alpha}\left(j_{\eta_i}(t)+j_{\varphi_i}(t)+j_{u_i}(t)\right)\right]< V_{\eta_i}(0)=\eta_i^T(0)P_{\eta_i}\eta_i(0)+\varphi_i^T(0)P_{\varphi_i}\varphi_i(0)+x_i^T(0)P_{x_i}x_i(0)=J_{x_i}^* \quad (95)$$

So, the upper bound $J_{x_i}^*$ of the quadratic guaranteed cost function can be obtained.

$$D^{\alpha}V_{\eta_i}(t)+j_{\eta_i}(t)+j_{\varphi_i}(t)+j_{u_i}(t)\leq \begin{bmatrix}\eta_i(t)\\ \varphi_i(t)\\ x_i(t)\end{bmatrix}^T \hat{\Sigma}_{\eta_i}\begin{bmatrix}\eta_i(t)\\ \varphi_i(t)\\ x_i(t)\end{bmatrix} \quad (96)$$

where

$$\hat{\Sigma}_{\eta_i}=\begin{bmatrix}\hat{\Sigma}_{\eta_{i_{11}}} & \hat{\Sigma}_{\eta_{i_{12}}} & \hat{\Sigma}_{\eta_{i_{13}}}\\ * & \hat{\Sigma}_{\eta_{i_{22}}} & \hat{\Sigma}_{\eta_{i_{23}}}\\ * & * & \hat{\Sigma}_{\eta_{i_{33}}}\end{bmatrix}$$

$$\hat{\Sigma}_{\eta_{i_{11}}}=(A_i+E_iC_i)^TP_{\eta_i}+P_{\eta_i}(A_i+E_iC_i)+\left(\alpha_{\eta_i}+\varepsilon_{\varphi_i}+\varepsilon_{x_i}\right)(W_i+K_i)^T\tilde{B}_i^T\tilde{B}_i(W_i+K_i)+\left(\alpha_{\eta_i}^{-1}+\beta_{\eta_i}^{-1}+\gamma_{\eta_i}^{-1}+\epsilon_{\eta_i}^{-1}\right)P_{\eta_i}M_iM_i^TP_{\eta_i}+\varepsilon_{\eta_i}^{-1}P_{\eta_i}E_iN_iN_i^TE_i^TP_{\eta_i}+\omega_{\eta_i}\theta_i^2I_n+\omega_{\eta_i}^{-1}P_{\eta_i}P_{\eta_i}+G_{\eta_i}+(W_i+K_i)^TG_{u_i}(W_i+K_i)$$

$$\hat{\Sigma}_{\eta_{i_{12}}}=-(W_i+K_i)^TB_i^TP_{\varphi_i}-(W_i+K_i)^TG_{u_i}W_i$$

$$\hat{\Sigma}_{\eta_{i_{13}}}=-(W_i+K_i)^TB_i^TP_{x_i}-(W_i+K_i)^TG_{u_i}K_i$$

$$\hat{\Sigma}_{\eta_{i_{22}}}=(A_0+B_0K_0+B_iW_i)^TP_{\varphi_i}+P_{\varphi_i}(A_0+B_0K_0+B_iW_i)+\left(\alpha_{\varphi_i}+\gamma_{x_i}+\epsilon_{\eta_i}\right)W_i^T\tilde{B}_i^T\tilde{B}_iW_i+\left(\alpha_{\varphi_i}^{-1}+\beta_{\varphi_i}^{-1}+\gamma_{\varphi_i}^{-1}+\varepsilon_{\varphi_i}^{-1}\right)P_{\varphi_i}M_iM_i^TP_{\varphi_i}+\omega_{\varphi_i}^{-1}P_{\varphi_i}P_{\varphi_i}+\omega_{\varphi_i}\sigma_i^2I_n+G_{\varphi_i}+W_i^TG_{u_i}W_i$$

$$\hat{\Sigma}_{\eta_{i_{23}}}=P_{\varphi_i}(A_i+B_iK_i-A_0-B_0K_0)+W_i^TB_i^TP_{x_i}+W_i^TG_{u_i}K_i$$

$$\hat{\Sigma}_{\eta_{i_{33}}}=(A_i+B_iK_i)^TP_{x_i}+P_{x_i}(A_i+B_iK_i)+\left(\alpha_{x_i}+\beta_{\varphi_i}+\beta_{\eta_i}\right)\tilde{A}_i^T\tilde{A}_i+\left(\beta_{x_i}+\gamma_{\varphi_i}+\gamma_{\eta_i}\right)K_i^T\tilde{B}_i^T\tilde{B}_iK_i+\left(\alpha_{x_i}^{-1}+\beta_{x_i}^{-1}+\gamma_{x_i}^{-1}+\varepsilon_{x_i}^{-1}\right)P_{x_i}M_iM_i^TP_{x_i}+\omega_{x_i}\theta_i^2I_n+\omega_{x_i}^{-1}P_{x_i}P_{x_i}+\varepsilon_{\eta_i}\tilde{C}_i^T\tilde{C}_i+K_i^TG_{u_i}K_i$$

By employing Lemma 6 and let $\boldsymbol{Y}_{\boldsymbol{\eta_i}}=\boldsymbol{P}_{\boldsymbol{\eta_i}}\boldsymbol{E}_{\boldsymbol{i}}$, we can obtain (84) and this completes the proof.

4. Simulation results

In this section, an example is presented to verify the applicability and effectiveness of the scheme proposed.

Consider a MAS with one leader and 6 followers, the directed interaction topology graph is depicted in Figure 2. The directed interaction topology graph contains a directed spanning tree with the leader rooted which can be seen obviously from Figure 2.

Hence, the matrix $\mathcal{H}$ is given as:

$$\mathcal{H}=\begin{bmatrix}2&0&0&-1&0&0\\0&2&-1&0&0&0\\-1&0&2&0&0&-1\\0&-1&0&2&-1&0\\0&0&-1&0&1&0\\0&0&0&-1&0&1\end{bmatrix}$$

Consider $\alpha=0.9$, $n=2$, $m_i=o_i=1;\forall i\in\bar{N}\cup\{0\}$ and the parameters of MAS are as follows:

$$A_i=\begin{bmatrix}0&1\\-a_2(i)&-a_1(i)\end{bmatrix},\ B_i=\begin{bmatrix}0\\b(i)\end{bmatrix},\ C_i=\begin{bmatrix}0&c(i)\end{bmatrix},\ f_i\left(x_i(t)\right)=\begin{bmatrix}0.1\sin\left(x_{i_1}(t)\right)\\0.1\sin\left(x_{i_2}(t)\right)\end{bmatrix}$$

$M_i=0.01I_2$, $N_i=0.01$, $H_i(t)=\cos(t)\,I_2$, $\tilde{A}_i=A_i$, $\tilde{B}_i=B_i$, $\tilde{C}_i=C_i$, $\theta_i=\sigma_i=0.1$

where $a_1=\{5,0,0,1,1,2,2\}$, $a_2=\{5,6,5,5,6,6,5\}$, $b=\{1,1,2,3,3,2,1\}$, $c=\{1,2,2,1,3,1,1\}$ for $i\in\{0,1,2,3,4,5,6\}$. $x_0(0)=\begin{bmatrix}10\\-10\end{bmatrix}$, $x_1(0)=\begin{bmatrix}6\\-14\end{bmatrix}$, $x_2(0)=\begin{bmatrix}14\\-6\end{bmatrix}$, $x_3(0)=\begin{bmatrix}13\\-13\end{bmatrix}$, $x_4(0)=\begin{bmatrix}7\\-7\end{bmatrix}$, $x_5(0)=\begin{bmatrix}9\\-14\end{bmatrix}$ and $x_6(0)=\begin{bmatrix}11\\-8\end{bmatrix}$. $G_{u_i}=0.05$, $G_{\eta_i}=0.1I_2$, $G_{\varphi_i}=I_2$, $G_{v_i}=0.1I_2$, $G_{\zeta_i}=0.1I_2$, $Q=0.1I_2$ $\rho=0.25$.

From the aforementioned theorems, by using the LMI toolbox in MATLAB, gain matrices can be obtained as the following.

$$E_i=\begin{bmatrix}e_1(i)\\e_2(i)\end{bmatrix},\ K_i=\begin{bmatrix}k_1(i)&k_2(i)\end{bmatrix},\ W_i=\begin{bmatrix}w_1(i)&w_2(i)\end{bmatrix},\ F_i=\begin{bmatrix}f_1(i)&f_2(i)\\f_3(i)&f_4(i)\end{bmatrix},\ T_i=\begin{bmatrix}t_1(i)&t_2(i)\\t_2(i)&t_3(i)\end{bmatrix}$$

where $e_1=\{-53.3,36.2,22.3,43.9,28.2,32.7,51.7\}$, $e_2=\{59.5,-19.4,-12.8,-27.7,-24.6,-27.4,-58.6\}$, $k_1=\{-1.21,-1.32,-0.87,-0.23,-0.23,-0.18,-0.12\}$, $k_2=\{4.3,-1.47,-0.92,-0.22,-0.22,-0.07,0.05\}$ for $i\in\{0,1,2,3,4,5,6\}$ and $w_1=\{-16.93,-12.83,-7.7,-10.99,-14.74,-9.5\}$, $w_2=\{-13.73,-11.24,-6.66,-7.71,-9.8,-8.79\}$, $f_1=$

$\{-0.38,-0.38,-0.49,-0.49,-0.51,-0.51\}$, $f_2 = \{-0.03,-0.03,-0.04,-0.04,-0.04,-0.04\}$, $f_3 = \{-0.05,-0.05,-0.07,-0.07,-0.07,-0.07\}$, $f_4 = \{-0.07,-0.07,-0.08,-0.08,-0.08,-0.08\}$, $t_1 = \{10.88,10.88,13.73,13.73,10.74,10.74\}$, $t_2 = \{0.98,0.98,1.24,1.24,0.93,0.93\}$, $t_3 = \{0.48,0.48,0.61,0.61,0.38,0.38\}$ for $i \in \{1,2,3,4,5,6\}$.

The pseudo-state trajectory $x_i(t)$, the consensus error trajectory $\delta_i(t)$, the observer error trajectory $\eta_i(t)$ and the control inputs trajectory $u_i(t)$ of the MAS are shown in Figure 3, Figure 4, Figure 5 and Figure 6, respectively. The event-triggered consensus control protocol $v_i(t)$ and the internal execution interval of followers is shown in Figure 7 and Figure 8.

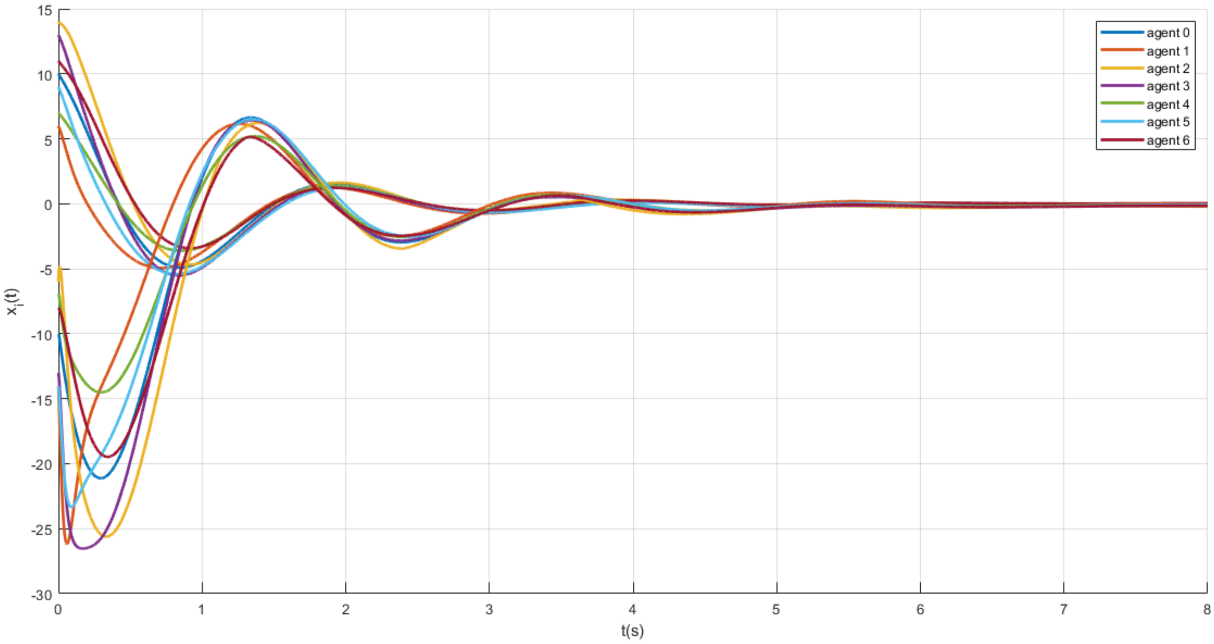

Figure 3: The pseudo-state trajectory $x_i(t)$ of the MAS.

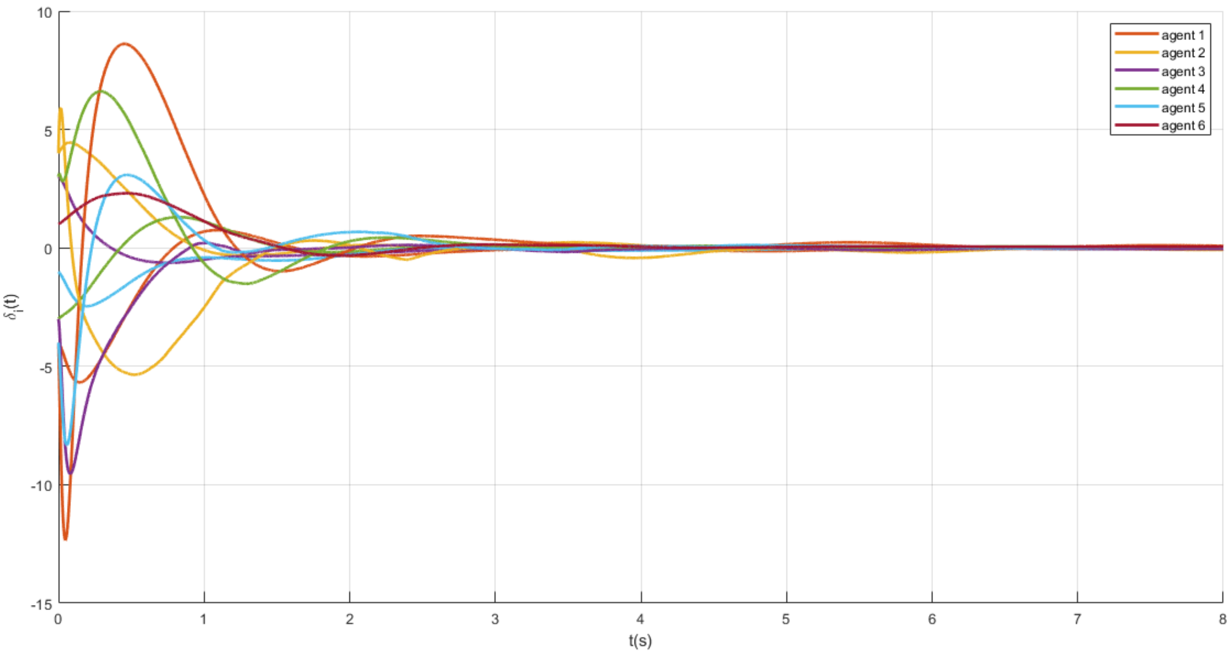

Figure 4: The consensus error trajectory $\delta_i(t)$ of the MAS.

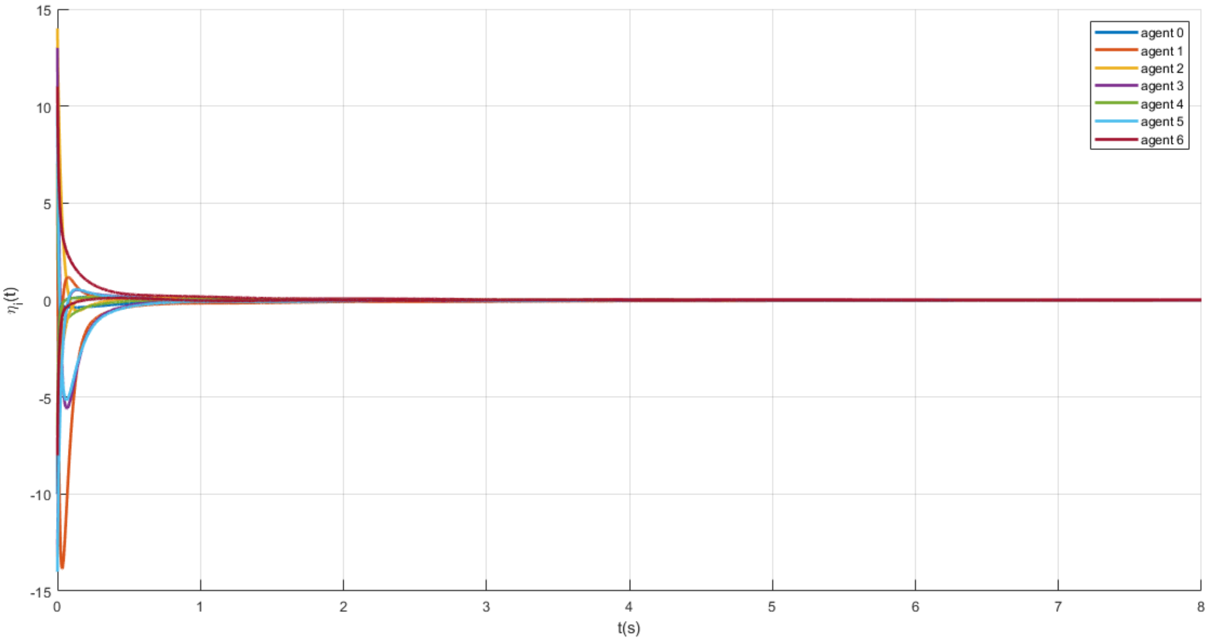

Figure 5: The observer error trajectory $\eta_i(t)$ of the MAS.

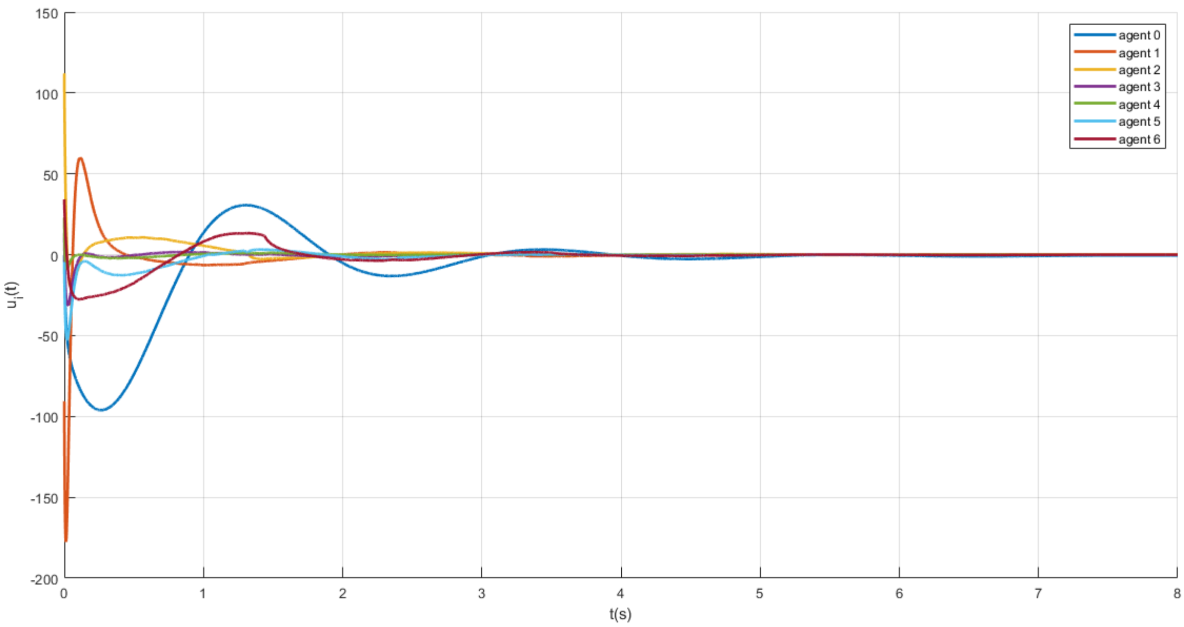

Figure 6: The control inputs trajectory $u_i(t)$ of the MAS.

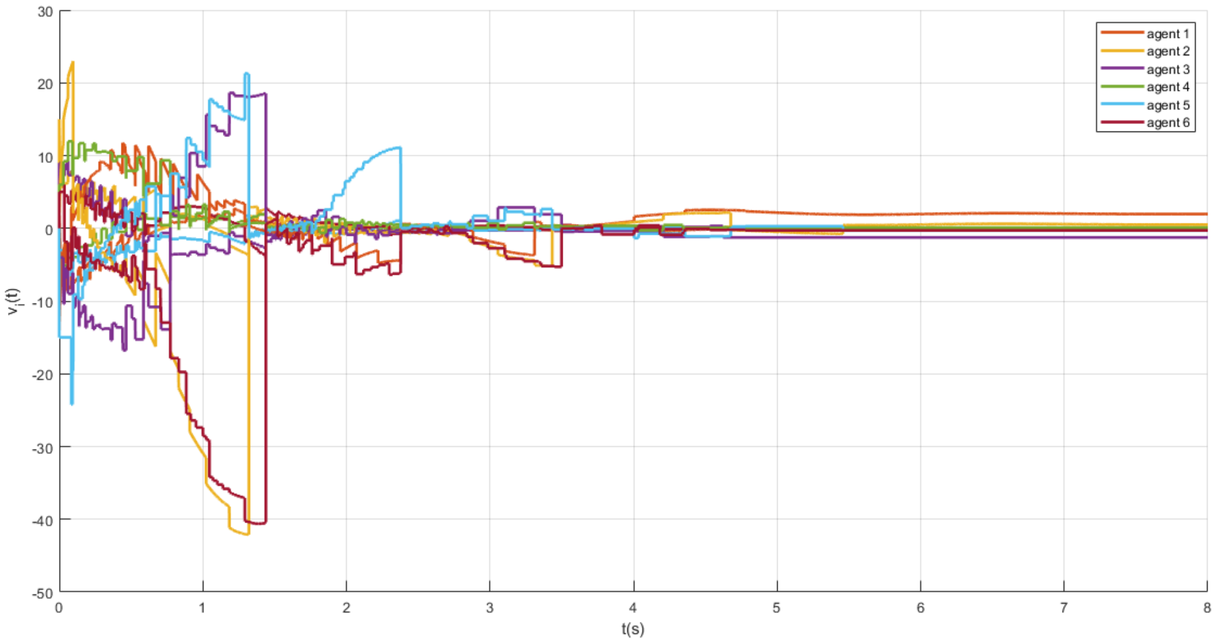

Figure 7: the event-triggered consensus control protocol $v_i(t)$ of the MAS.

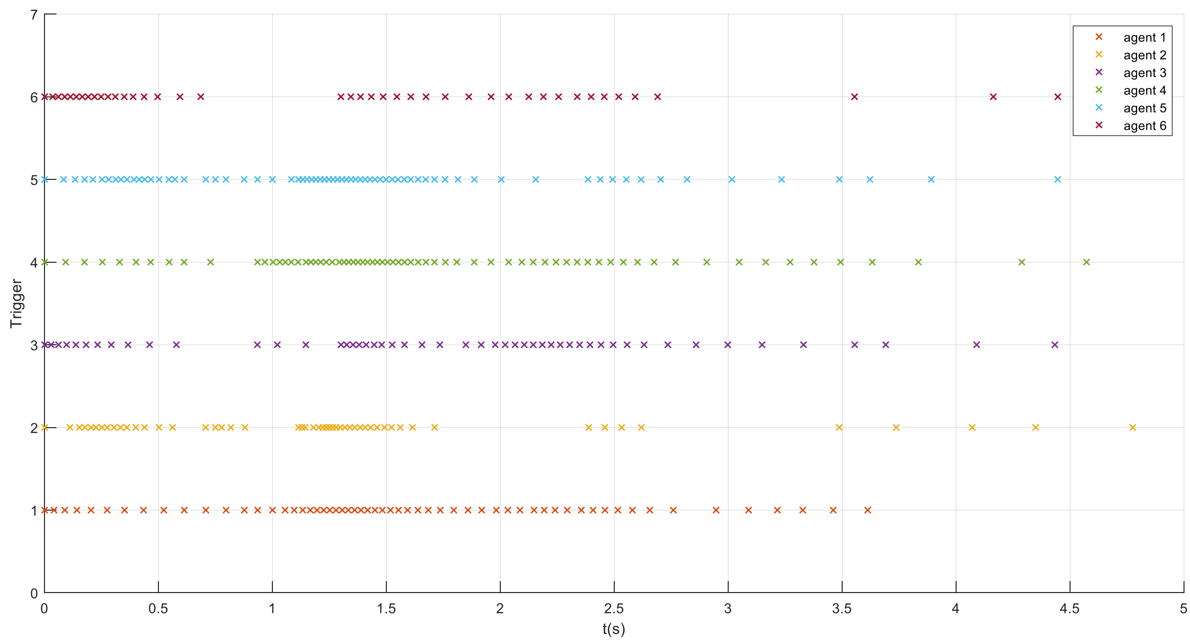

Figure 8: Internal execution interval of followers.

According to the figures, it can be seen that observers estimate pseudo-states of agents correctly and the leader-following consensus has been achieved properly while the communication between the followers is based on event-triggered strategy. Updates to the control protocol occur only upon the satisfaction of the triggering condition but the control inputs of agents change continuously.

5. Conclusion

In this paper, the event-triggered leader-following GCC for heterogeneous uncertain nonlinear FOMASs based on observers has been studied. It considered that each agent has different fractional-order dynamics with state, input, and output uncertainty. For saving communication resources, an event-triggered strategy proposed that control protocol doesn't update until triggering condition maintain. Based on the fractional Lyapunov direct method and the proposed event-triggered strategy, problem described by LMIs and some criteria were obtained to ensure that GCC was achieve. To show the effectiveness of the proposed method a numerical simulation is given and the results are reported. Future works will focus on the fixed-time consensus leader-following consensus for heterogeneous uncertain nonlinear FOMASs based on observers.

Declaration of Competing Interest

Authors declare that they have no conflict of interest.

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