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Modeling Time-Delay in Consensus Control: A Review

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ABSTRACT

A multi-agent system (MAS) consists of spatially distributed agents that data exchanges between them through a communication network. A protocol among the agents to reach the consensus is designed by cooperative control technique. Generally, time-delay is a challenge of control systems. Specifically, the time-delay problem is receiving more attention in the cooperative control of MASs because of the more probable time-delay in data transmission among the agents. Time-delay reduces the control effectiveness by deteriorating stability and the performance of the control protocol. These drawbacks demand careful time-delay consideration in cooperative control of MASs. In this paper, we carry out a survey on the methods for mitigating time-delay impacts in cooperative control of MASs by reviewing about 120 papers from 2013 to 2024. Time-delay can result from natural/unintentional causes such as limited communication bandwidth, packet dropout, or overhead in communicating and from cyber-attack/intentional causes that flood the communication network. Moreover, we classify the methods based on the location of occurrence and the type of delay. In the end, we present our perspective on upcoming research directions.

1. Introduction

The goal of cooperative control is to assign a control protocol for each agent to guarantee the desired behavior of the entire system and achieve consensus. The distributed control protocol ensures that all states converge to the same constant steady-state value, known as the consensus value. Recently, cooperative control of MAS has garnered increased attention with various engineering applications, such as, but not limited to, distributed sensor networks, cooperative control of unmanned aerial vehicles (UAVs), smart grids, formation control, and flocking [1]. The cooperative performance of a group of independent agents brings several advantages, including scalability, flexibility, adaptivity, and low cost of operation in comparison to separate control of individual agents if the quality of data exchange through a communication network is guaranteed [2]. In real applications, receiving information from neighbors through a communication network may be delayed. The primary causes of data transmission delays are packet loss within wireless communication networks, imprecise sensor measurements, and errors in sampling.

In general, time-delay corresponds to a situation in which an intentional or unintentional shift occurs in a signal which frequently occurs in various engineering processes and affects their inner dynamics. From the control engineering perspective, delays can cause instabilities and make it harder for the system analysis, controller design, and state estimation. Delay may cause a periodic oscillation or any other chaos in MASs [3]. By preventing the correct decision at any time. delay causes a reduction in system performance or stability. To tackle these disadvantages, special attention must be paid and necessary compensation should be applied in the

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designation of process, actuators, sensors, and also control protocols.

In recent years, researches on time-delay in MAS systems has been increased significantly in a way that the total number of published papers from 2018 to 2020 is more than 40% of the total papers since 2013. Fig.1 shows and compares the percentage of time-delay papers in MASs for each year from 2013 to 2024. In this figure, the ratio of the number of the published papers in each year to total MAS papers has been shown. It is understood that in 2018 to 2020 the number of research works published in this area has been increased significantly which shows researches on time-delay in MASs is continuing and increase with the growth of MASs application.

The objective of this paper is to review time-delay compensation in cooperative control of MASs. To this end, we have reviewed more than 120 papers from 2013 to 2024. Based on the reviewed literature, we have classified time-delay in MASs in two categories: First, based on the location of occurrence. The delay in time can happen at the input [4], output [5], and sampling [6]. The second category is based on the type of time-delay that can be fixed [7], time-variable [8] including time-varying delay, and switching time-delays.

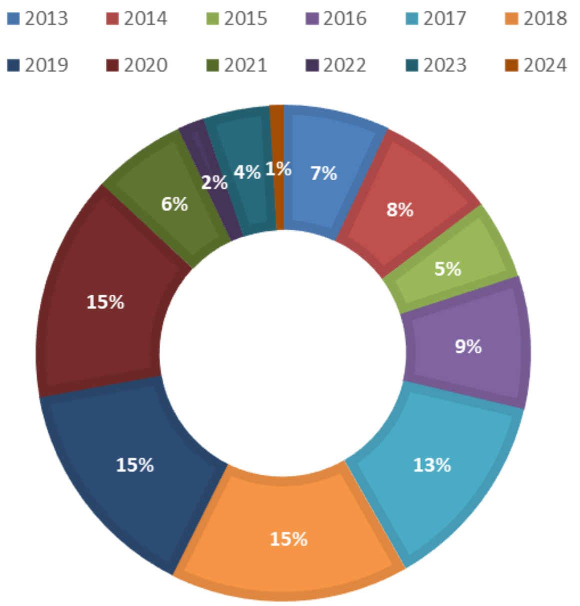


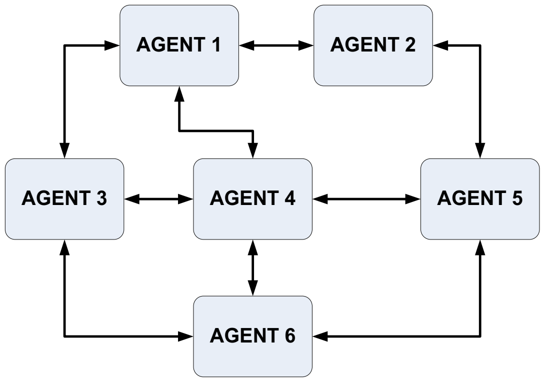
Fig. 1. The percentages of published papers in different years

The structure of the paper is designed as outlined below: After providing introduction, in section I, in section II, the required definitions and concepts are provided concisely. In section III, and IV, the mentioned categories based on the location of occurrence, and the type is provided, respectively. In section V, time-delay in cooperative control of practical MASs is studied by bringing a few examples. A future perspective and conclusion have been brought in VI.

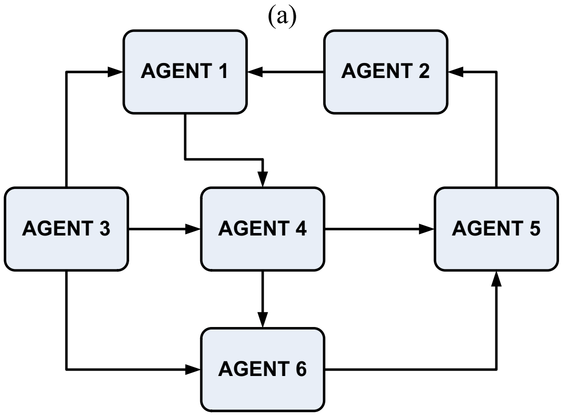
2. Preliminary

MASs consist of a few agents that operate as a group to approach a specific goal. This goal can take many forms such as consensus, flocking, frequency matching, synchronization, formation, and rendezvous. These agents connect to together by a communication graph. Generally, a graph is shown as a pair $GE = (VE, ED)$ that $VE = \{ve_1, ve_2, \dots, ve_N\}$ and N is number of vertices or nodes and ED is a set of arcs or edges. Also, ED are denoted as (ve_i, ve_j) which is termed an arc or edge from ve_i to ve_j, and depicted as an arrow with its head at ve_j and tail at ve_i. a_{ij} is a positive weight that associate with each edge (ve_i, ve_j) and connectivity or adjacency matrix is $AD = [ad_{ij}]$ [1]. If $(ve_i, ve_j) \in E \Longrightarrow (ve_j, ve_i) \in E$, $\forall i, j$ the graph is bidirectional, otherwise it is a digraph or directed graph and it is said undirected if $ad_{ij} = ad_{ji}$, $\forall i, j$, as shown in Fig. 2.

In a consensus control problem, it is necessary to determine a control protocol that directs all states to the same constant steady-state values ($X_i = X_j, \forall i, j$) which is named as a consensus value [1].



(a)



(b)

Fig. 2. (a). bidirectional graph , (b). digraph

Throughout the paper, Laplacian matrix is used frequently. The weighted in-degree of node ve_i is defined as the sum of the elements of the i-th row of AD, $d_i = \sum_{j=1}^{N} ad_{ij}$ [1]. Let us define in-degree diagonal matrix as $DD = diag[dd_i]$. The Laplacian matrix of the MAS is defined as $LL = DD - AD$. By using the Laplacian matrix, the overall state-space representation of MAS is as $\dot{X} = -LL$ [7]. Since LL is a symmetric matrix i.e. $LL = LL^T$, all eigenvalues of the matrix LL are real, we can arrange them as $\lambda_1 \leq \lambda_2 \leq \cdots \leq \lambda_N$. Consensus speed in the MAS depends on the second eigenvalue λ_2 (Fiedler Eigenvalue) of the graph LL. A graph with a larger $\lambda_2$value converges in less time [2].

According to [9], a simple consensus protocol is defined of the state of integrator agent dynamics.

This simple dynamic is used for all MASs in this review.

$$\dot{x}_i = u_i \tag{1}$$

that i represent the agents, and for a fixed or switching topology and without time-delay, the consensus protocol is written as follows:

$$u_i(t) = \sum_{j\in N_i} ad_{ij}\left(x_j(t) - x_i(t)\right) \tag{2}$$
$$i \in \{1, \dots, N\}$$

where N_i is a set of neighbors of the agent.

Another operator in this paper is $\otimes$ that denote the Kronecker multiplication. The Kronecker multiplication is an operator between two matrices A and B with an arbitrary dimension that the result is a block matrix.

$$A \otimes B = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix} \tag{3}$$

3. Location of time-delay delay occurrence

In this section, time-delay is classified based on the location of occurrence in MASs. Some delays may happen in the communication path as such input and output delay, but some delays are due to internal structure of agents such as feedback, and sampling delay. These delays are shown in Fig. 3.

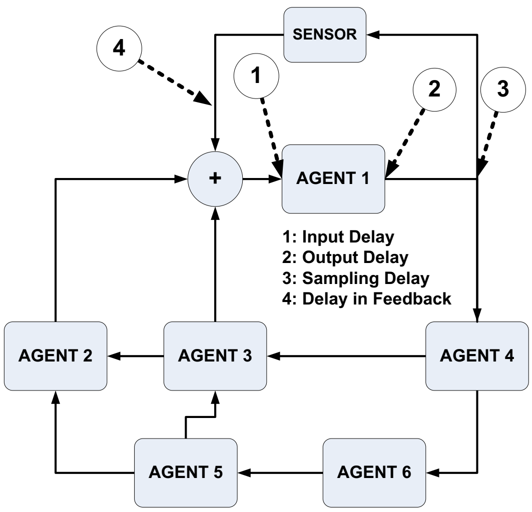


Fig. 3. Time-delay in a MAS

In [10], consensus of MASs with diverse communication and input delays based on the analysis of frequency-domain has been studied. For the systems based on undirected graphs and directed graphs with communication delays, two consensus protocol conditions are obtained separately. In the mentioned paper, under diverse communication delays, the consensus protocol becomes:

$$u_i(t) = \sum_{j\in N_i} ad_{ij}\left(x_j(t - T_{ij}) - x_i(t)\right) \tag{4}$$

where represents the communication time-delay from agent to agent and is the adjacency elements of in the digraph $GE = (V, ED, AD)$.

3.1. Communication Delay

3.1.1. Delay in Input

As shown in Fig.3, delay can happen in the input of the agents and it has been extensively studied in previous papers. In MASs with input delay, state equations are modeled as follows [7]:

$$\dot{x}_i(t) = \sum_{j\in N_i} ad_{ij}\left(x_j(t - T) - x_i(t)\right) \tag{5}$$
$$i \in \{1, \dots, N\}$$

where T represents time-delay. In this study, a margin for the time-delay is determined, which represents the maximum input delay for which the system can still maintain consensus using the same protocols. For time-delay which is less than the margin, consensus is guaranteed. The margin decreases as the largest eigenvalue of the Laplacian matrix of the system increases. [11]. This method has been also used in [3], where the focus is on achieving consensus and introduces quasi-consensus in multi-agent dynamical systems. In [12], for systems with a single unstable open-loop pole, the delay margin has been obtained. It has been demonstrated that in scalar discrete-time systems, a positive maximum delay margin for achieving consensus exists only if the open-loop pole of the system falls within a particular range $\left(-1{,}1 + \frac{\lambda_2}{\lambda_N}\right)$. Also, a bound for input delays has been established for leader-following consensus of heterogeneous fractional-order MASs in [13], In these systems, a group of agents as leaders drives the group of followers to achieve consensus. In [14], An H_2 controller in frequency domain is used to MAS control and determined a margin for delay time. Also, in [15], the study explores consensus in leader-follower nonlinear MASs with input time-delay. It introduces a time-varying gain method to formulate consensus protocol and achieves delay-independent leader–follower consensus by constructing a Lyapunov–Krasovskii functional.

Use estimators is another way to compensate for the effect of delay in MASs. In [16], a truncated prediction feedback method has been applied for achieving consensus control in a Lipschitz nonlinear MAS with input delay. Sufficient conditions for consensus guarantee were derived using Linear Matrix Inequality (LMI). The LMI method was first introduced by Lyapunov, with the Lyapunov inequality representing a specific instance of LMI. In [17], A method based on asymptotic behavior approximation has been developed for analyzing a class of MASs with communication delay. The outlined estimation technique involves a delay-free MAS with a weighted consensus protocol, yielding results comparable to the original system and effectively well estimates the dynamic behavior of the main system with telecommunication delay. By utilizing the predictor and fine-tuning its gain, it becomes feasible to design a trajectory tracking consensus protocol suitable for heterogeneous MASs [18]. In [19], two-state and finite-time estimators are employed for target tracking and delay estimation in distributed tracking control of Lipschitz nonlinear MASs under external disturbances and input delay conditions. The state estimators reconstruct the system's state based on the measured output using a model description of the system [20]. A finite-time estimator acts as an observer crafted to precisely estimate the leader's state information within a limited time period [21]. Additionally, in [22] consensus has been achieved for a class of agents facing input delay and external disturbances through the implementation of two consensus protocols utilizing a future state predictor and a finite time estimator. In [23], a distributed disturbance observer for consensus. consensus control has been used for each agent that can reject disturbance and delay effect. The estimation is based on the relative state information obtained through the communication network. Also, in [24], the states of the leader for each follower in a second-order Multi-Agent System (MAS) have been successfully determined using a fixed-time distributed observer. Nevertheless, delays can extend the convergence time, requiring larger control gains to reach consensus. When the constant input delay is unknown, an estimation technique is applied to determine the delay, followed by a transformation that converts the time-delayed system into a second-order system without delays. The study of H_∞ consensus for linear multi-agent systems on general directed graphs, considering constant input delays and external disturbances, has been addressed in [25]. In this paper, the concept of H_∞ consensus is introduced. A finite-dimensional component of the classical state predictor is used to approximate a truncated feedback prediction of the agent's state during the delay period. H_∞ consensus is supported by the properties of the Laplacian matrix, and the feedback gain is designed using an iterative process involving linear matrix inequalities. Also, a distributed H_∞ observer is employed to acquire state information from both the agent and its neighboring agents in [26], then a distributed controller is designed using a Lyapunov–Krasovskii functional (LKF) and delay-dependent region partitioning. The research defines a sufficient condition for the controller using Linear Matrix Inequalities (LMI).

The event-triggered consensus algorithm also has been used to MASs control with delay. In [27] and [28] the event-triggered consensus for Multi-Agent Systems (MASs) with the occurrence of input delay to avoid Zeno behaviour has been developed. Furthermore, a control scheme utilizing an observer has been implemented to handle the problem of unobservable states. An event-triggered approach is employed to minimize communication overhead, updating each agent's control input only when a predefined trigger condition is satisfied.

The event-based control algorithm for managing the distributed control of multi-agent systems (MASs) with input time delays between the controller and actuator in leader-following consensus was developed by [29]. This algorithm updates the controller based on events, with updates occurring solely at specific event times. The study provides both necessary and sufficient conditions to ensure consensus, whether or not continuous communication between neighboring agents is maintained, and also addresses the Zeno behavior of the triggering time sequences. In [30], an event-triggered control algorithm is introduced to enable distributed containment control for multi-agent systems (MASs) characterized by high-order dynamics and input delays. Each follower detects events through its control input, enabling them to asymptotically converge into the convex hull formed by the leaders. So, in [31], a distributed event-triggered strategy is proposed for uncertain nonlinear multi-agent systems with time delays. Criteria have been established to ensure guaranteed cost consensus using the Lyapunov method. Additionally, a self-triggered algorithm for leader-following consensus in general linear multi-agent systems with input delays was introduced in [32], utilizing matrix theory, algebraic graph theory, and stability theory. Consensus can be achieved asymptotically, and the Zeno behavior of triggering time sequences can be avoided. For a linear multi-agent system with input time delay, an observer-based bipartite control technique incorporating a self-triggered mechanism was proposed in [33]. Similarly, [34] also employed a self-triggered approach. The self-triggered algorithm determines the next sampling time based on local information and a threshold condition from the previous trigger.

Some studies have utilized model reduction techniques to transform multi-agent systems (MASs) with delays into models without delays, thereby eliminating the impact of delays on system stability. In [35], a data-driven approach for distributed optimal consensus control of unknown MASs was explored, with the system model converted to a delay-free form using model reduction. Optimal consensus control algorithms were derived for the delay-free system based on coupled Hamilton-Jacobi equations and Bellman's optimality principle, facilitating the proposed data-driven adaptive dynamic programming approach. In [36], robust consensus control for MASs with uncertain parameters was achieved through the Artestein model reduction method and state transformation. Meanwhile, [37] designed distributed optimal consensus control for continuous-time heterogeneous linear MASs by first converting the input-delayed system to a discrete-time delay-free system via discretization and model transformation, and then developing an optimal control policy for each agent using Hamilton-Jacobi-Bellman (HJB) equations.

In [38], a leader-following consensus protocol for heterogeneous multi-agent systems (MASs) with first-order and second-order agents experiencing time-varying and input delays was analyzed using Lyapunov-Krasovskii functions. The study derived sufficient consensus conditions in the form of linear matrix inequalities (LMIs) for systems with a fixed interconnection topology. Similarly, [36] and [23] employed Lyapunov–Krasovskii functions to establish sufficient conditions for consensus. The Lyapunov–Krasovskii functional is a vital tool for the stability analysis of closed-loop time-delay systems, as discussed in [39], and its applications are further elaborated in [40]. Additionally, in [41], Lyapunov–Krasovskii functionals are crucial for analyzing nonlinear MASs with input delays and potential false-data-injection (FDI) attacks. A neural network (NN) is used for FDI attack detection, and

Lyapunov-based stability analysis is employed to ensure semi-global uniformly ultimately bounded tracking.

In [42], consensus for second-order multi-agent systems (MASs) with nonlinear dynamics was examined, with consensus criteria established using algebraic graph theory, matrix theory, and Lyapunov stability methods. Similarly, [43] provided a rigorous proof that all agents in nonlinear MASs with input delays can achieve consensus, based on Lyapunov stability theory. [44] explored dynamic models for linear and nonlinear coupling in MASs with antagonistic interactions and communication delays, proposing a bipartite consensus solution for both linear and nonlinear coupling networks. Their results, derived from Lyapunov stability theory and other mathematical analyses, confirmed that all agents in signed networks can achieve consensus on a value, except for the sign. Stability in this context refers to the ability to maintain consensus under varying conditions. In [45], the focus was on achieving consensus in islanded microgrid networks of distributed generators amidst communication noise and delays. The aim was to regulate output voltage, address frequency deviations, and ensure accurate real and reactive power-sharing among distributed generators. This protocol used rigorous Lyapunov analysis to demonstrate mean square consensus restoration, enhancing system reliability and reducing sensitivity to failures. In [46], a simple quadratic Lyapunov function was used to assess consensus, presenting several algebraic criteria for leader-following consensus in fractional-order nonlinear MASs. [47] applied signed graph theory, the fractional Razumikhin technique, and a common Lyapunov function approach for controlling nonlinear fractional MASs. Lastly, [48] introduced an observer-based fuzzy adaptive controller for nonlinear MASs, utilizing Lyapunov function theory in the design of the fuzzy adaptive controller. They also examined the stability of the closed-loop system and the conditions necessary for achieving consensus.

[49] investigated an asynchronously-coupled consensus algorithm for second-order multi-agent systems (MASs) with input delays, demonstrating asymptotic consensus of the agents through frequency domain analysis. Similarly, [50] addressed consensus issues in first-order MASs with multiple input delays within a directed network. Their study covers continuous, discrete, and continuous systems with a proportional plus derivative controller. Stability was analyzed using the generalized Nyquist criterion and a frequency-domain approach. The system's convergence primarily depends on each agent's input time-delay and adjacent weights, while being independent of the communication delay between agents, regardless of whether the system is continuous or discrete. Also in tracking consensus problems, the consensus stability has been proved by the frequency response approach [51],[18].

In [52] a delay consensus margin is calculated for a first-order MAS with an undirected graph using feedback protocols to ensure system stability even with the maximum delay range.

3.1.2. Delay in Output

Output delay is the second branch of delay categorization based on the location of time-delay that occurs at the output of the agents. The considered articles in this category have used order reduction, observer, reduced-order observer, and event-trigger to reach consensus and used frequency domain analysis to derive consensus condition. The reduced-order method has been used in a H_∞ consensus for heterogeneous MASs with first-order and second-order integrator agents in [53], and the estimated delay time has been obtained using the LMI method. In [54] the event-trigger strategy along with reduced-order observer to obtain consensus of MAS with output time-delay has been used. Sufficient condition for the consensus of MASs has been obtained by using the integral inequality technique and matrix theory. Also, in [55], the event-trigger tracking consensus protocol in linear MASs with output delays has been obtained By Lyapunov stability theory and aimed removing delay effect and Zeno-behavior.

Despite the challenges posed by delays in MASs, in [56], explores the beneficial impact of time-delay on achieving consensus in second-order MASs. Closed-loop systems lacking intentionally induced delays struggle to reach consensus with a non-delay position-based protocol. Hence, the introduction of a specific delay value is crucial for enabling system consensus.

3.1.3. Delay in Input/Output

The third category of articles, deal with the cases that delays occur at both input and output and we investigated them in this section. In [57] has been introduced two factors to Input/output delays MASs with non-uniform and asymmetric (heterogeneous) time-delays: first, time-delays are caused by communication transmission on networks and second when the computing ability of agent is not high enough and cause to computing delay.

Network topology plays an important role in reaching to consensus and this topic has been investigated in [58] and shown it is necessary that a simple eigenvalue of the Laplacian L be zero to achieve consensus, in other hand the network has a spanning tree. In this article the network G is undirected, and then the Laplacian L has real eigenvalues, which can be ordered as $0 = \lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n \leq 2$. The smallest eigenvalue λ_1 is zero and $\lambda_2 > 0$ if and only if the graph is connected. The largest eigenvalue is 2 if and only if a connected component of GGG is both bipartite and non-trivial. Also, in [59], has been proved the presence of a spanning tree is essential. In [60] the consensus condition has been derived based on agent dynamics, communication delay, and the Eigen ratio $\left(\lambda_2/\lambda_N\right)$ of the network topology for continuous-time first-order MASs with uniform constant communication delay. And in [61] the Eigen ratio $\left(\lambda_2/\lambda_N\right)$ determine allowable delay bound. The delay bound is maximized when the network topology G is complete and $\lambda_2 = \lambda_N$.

When delay is both input and output, the state equation is as follows for $I = 1,2,\ldots,N$: [62]

$$\dot{x}_i(t) = u_i\left(t - T_i^{in}\right) \tag{6}$$

$$y_i(t) = x_i(t - T_i^{out}) \quad (7)$$

The control protocol is obtained as

$$u_i(t) = -\sum_{j\in N_i} a_{ij}\big(y(t) - y_i(t)\big) \quad (8)$$

In this paper, by the state predictor controller as follows MASs with integrator dynamics has been controlled

$$u(t) = -k(L(G)\otimes I_{n)}\hat{y}(t) \quad (9)$$

$$\dot{\hat{y}}(t) = u(t) - k_p(\hat{y}(t-2T) - y(t)) \quad (10)$$

If the predictor is aware of the system's initial states, the prediction error consistently remains zero. Also, in [63] a predictor-based controller received discrete-time neighbor information and estimated the continuous-time neighbor state and obtained to bounded consensus.

In [64] sufficient conditions have been obtained by the Lyapunov function for linear MAS with stochastic sampling. Additionally, [65] demonstrated that consensus tracking can be achieved with asymptotic stability using algebraic graph theory and the Lyapunov–Krasovskii functional method.

3.2. Delay in Sampling

Sampling delays can lead to oscillations or divergence in the system, making it crucial not to overlook the impact of delays on the stability of MASs. In most articles in this category the delay decomposition technique has been used to obtain consensus and stability analysis has been done using different methods. In [66] sufficient and necessary conditions have been established to ensure bounded consensus tracking for second-order MASs. These conditions depend on factors such as the communication network's topology, control parameters, sampling delay, and sampling period. The derivation of these conditions involved utilizing techniques like probability limit theory, the augmented matrix technique, algebraic graph theory, and other methods to guarantee mean square bounded consensus tracking [67].

3.3. Delay in state and Feedback

Delay in state and feedback are also addressed in another set of articles that we describe below.

In [68], has been found an upper and lower bound for input delay. State time-delay of nonlinear MASs has been investigated and consensus protocol has been obtained using an adaptive neural network (NN) and the Lyapunov-Krasovskii function in [69]. In [70], the study explored the robustness of consensus protocols in sensor networks against various types of feedback delays with heterogeneous agent dynamics, and established consensus conditions using the Nyquist criterion. In [71] Through the utilization of open-loop and closed-loop iterative learning control, consensus is achieved in fractional-order MASs with state-delays. The controllers adjust the control law in each iteration to converge towards the optimal control law. Sufficient conditions for both control strategies are established through rigorous analysis. In [72], it was examined the stabilization problem for general first-order multi-agent systems with state time delays. Their analysis of distributed proportional-integral-derivative (PID) controllers begins by establishing the range of proportional gains needed to ensure stability for the distributed PID controller. Subsequently, within this range, the stabilizing region for integral and differential gains is derived. Furthermore, [73] provide a stability condition for multi-agent systems (MASs) with state time delays, deriving it from the upper bound solution of the continuous coupled algebraic Riccati equation (CCARE).

4. Classification based on type of delay

Another aspect of the categorization of the delay has been the type of delay. Delay may be fixed or time variable, although in practical systems we rarely have fixed delay, in many types of research, it has also been studied.

4.1. Fixed Delay

As in the previous sections, in the articles reviewed of this category use a variety of methods to reach consensus. Let's start with the network topology effect on reaching consensus. In [74], in nonlinear MASs has been determined a permissible bounded delay dependent on Laplacian eigenvalue that the system can reach consensus in this bound. also in [75], to achieve consensus, it has been obtained a relationship between the maximum admissible delay range and maximum degree of the network topology. This article applies the Lambert W function to evaluate the worst-case convergence rate of the algorithm. In [76], the eigenvector-eigenvalue method has been used to determine the maximum allowable delay bound and in [77], first, the maximum delay on any link has been determined and then obtained the MAS convergence rate based on maximum delay.

Using the independent network topology estimator is another method mentioned in [78], in this method the future states of the agents bounds and increase the maximum time-delay by a predictor-observer estimate. Also, in [62], a state predictor has been used to compensate the effect of fixed delay in the input and output is investigated. And in [79], a robust estimator has been used to delay compensation in a leader-follower MAS. Fig. 4 shows that delay compensation block diagram in this system.

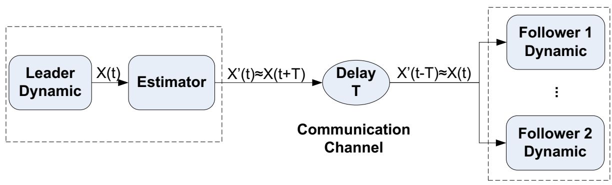


Fig. 4. Delay Compensation Block Diagram in Leader-Follower MASs [79]

[80] utilized the Lyapunov-Razumikhin theorem to develop an auxiliary system for mitigating delay effects. This theory, also discussed in [40], is used to establish the stability of systems with delays.

The fractional-order approach better captures the system's inherent characteristics in complex environments. In [81], the focus was on achieving consensus in fractional-order multi-agent systems

(MASs) using state-derivative feedback and fixed delays. The study determined two types of maximum tolerable delays necessary to ensure consensus within the system. Stability analysis and necessary and sufficient conditions to reach consensus are other goals of researchers. In [7] by Lyapunov-Krasovskii techniques given in terms of LMI, system stability has been evaluated.

In [82], has been used LMI and Lyapunov methods to stability analysis in a sliding mode controller for a MAS with uncertainty. The Lyapunov method in [74] has been used to converge analysis in leader/ follower nonlinear MASs. The same method has been used in [83] in general linear MASs. In [9], the Lyapunov function has been used alongside graph theory and matrix theory in order to consensus analysis.

In [76], analysed the obtained consensus, by eigenvector-eigenvalue method for a network of second-order agents with fixed topology and time-delay, by using frequency domain method.

In [84.] An adaptive dynamic programming (ADP) approach is utilized to achieve consensus in a discrete-time (DT) tracking control scenario for MASs with time delay. The method ensures the fulfillment of necessary and sufficient conditions, and an error estimator is introduced to assess the tracking error of the MASs based on input and output data.

4.2. Time Variable Delay

Variable time-delay is the last category in delay.

In switching topologies with time-varying delays, the control protocol is as follows:

$$u_i(t) = \sum_{j\in N_i} a_{ij}\left[x_j\left(t-T_{ij}(t)\right)-x_i\left(t-T_{ij}(t)\right)\right] \tag{11}$$

$$\dot{x}_i(t) = \sum_{v_j\in N_i} a_{ij}\left[x_j\left(t-T_{ij}(t)\right)-x_i\left(t-T_{ij}(t)\right)\right] \tag{12}$$

Same as previous sections, network topology has an important role in reach consensus. Permitted delay bound has been determined by system topology specifications to ensure consensus is obtained ($T \in [0, \pi/2\lambda_{max}(L)]$). it has been proved if the network be directed, even with a long delay, the system will achieve consensus [8]. Also, in [85], it is proved that despite a spanning tree in the high-level heterogeneous MAS, all agents will converge. And in [86] has been emphasized the need for spanning tree to reach consensus in presence of delay.

Also, using estimator/observer-based controllers is one way to compensate for the time variable delay effect. In [87], has been used an estimator to Control of followers in non-affine nonlinear MASs with heterogeneous and completely unknown. Also, in [88] and [89] the distributed observers are used to estimate the required information in order to reach consensus. In [90] an observer base consensus protocol was proposed for consensus tracking problem in a nonlinear MAS with time delay. cause to unmeasurable agents' dynamics a nonlinear observer has been introduced to estimate the unmeasurable states and time delay compensated with a Lyapunov-Krasovskii functional. An estimator-based non-fragile tracking controller is presented for MAS with communication delay [91]. This controller uses the Lyapunov-Krasovskii functional. A low-gain nonlinear observer is employed to estimate the varying input delay in [92], followed by the design of an auxiliary system to produce delay compensation signals. Additionally, a distributed adaptive composite NN dynamic surface control is suggested to enhance tracking accuracy.

The feedback control law is another method to obtain consensus in time-delay MASs. In [89], dynamic output feedback control law alongside observer is used to reach consensus. In [93], [94] and [15], the state feedback law has been used. In [95], research has focused on achieving consensus in uncertain nonlinear MASs with a fixed and direct communication topology. By employing a stochastic variable based on a Bernoulli distribution, probabilistic time-varying delay information is transformed into deterministic time-varying delay with stochastic parameters. Utilizing the properties of the Laplacian matrix and implementing a state feedback controller, the consensus issue is addressed by evaluating the conventional stability of the closed-loop system. Similarly, [96] applied a Bernoulli random process to achieve mean-square robust consensus in MASs with second-order mobile agents, accounting for variable delays, noise, and packet losses.

Determining the allowable delay bound is a way to achieve consensus in the delayed MAS. In [97] a strictly positive lower bound for leader-following consensus along with the use of hybrid event-triggered control exclude Zeno behavior and create consensus. Also, in [98], is achieved an event-triggered consensus for a MAS with random nonlinear dynamics and time-varying delay and in [99] a delay-dependent triggering mechanism has been used for tracking consensus protocol in linear MASs. A consensus algorithm has been designed based on the maximum allowable delay on any link in [77] and [44].

In the time-variable delay MASs, to stability analysis and determine consensus necessary and sufficient conditions, have been used linear matrix inequality (LMI) and Lyapunov-Krasovskii methods and Other methods, including Lyapunov-Razumikhin, stochastic analysis, and the Lyapunov stability theory are less commonly used.

In [100], a robust integral sliding mode controller has been designed to consensus tracking control for leader-following MASs with higher-order and Stability analysis has been performed using LMI method. [101] established the convergence conditions for leader-following tracking consensus in high-order nonlinear dynamical multi-agent systems (MASs) with switching topologies and communication delays in noisy environments. Additionally, [39] conducted consensus analysis using linear matrix inequalities (LMIs). [102] derived sufficient conditions for achieving asymptotic mean-square consensus in distributed multi-agent systems (MASs) considering external disturbances and network imperfections, such as communication delays and random packet dropouts, using linear matrix inequalities (LMIs).

In [103], by employing Lyapunov-Krasovskii function and matrix by assigning LMI to the dynamics of each agent has been done stability analysis. Also, in [104]

Sufficient conditions on consensus of networked MASs with second-order dynamics were achieved by a discretized Lyapunov- Krasovskii functional method if a set of linear matrix inequalities with two parameters are feasible. In [105] the Lyapunov-Krasovskii functional has been constructed and has been used the linear matrix inequality theory to convergence analysis. In [106] the finite-time consensus of linear MASs has been investigated and Sufficient conditions for solving this problem were established through the application of Lyapunov-Krasovskii stability theory, algebraic graph theory, and integral inequalities. The algebraic graph theory has been used to stability analysis of Mean square average consensus for MASs in [107]. In [95] and [108] the stability conditions have been extracted using the Lyapunov-Krasovskii functional (LKF). In [109] the convergence analysis has been obtained by the Lyapunov-Razumikhin theorem alongside LMI conditions and Lyapunov-Razumikhin theory in [22] has been used to consensus analysis. Lyapunov theory is also applied in [94] and [43]. In [110], a study on high-order nonlinear multi-agent systems (MASs) is presented, where a fully distributed consensus protocol is developed to guide all agents towards achieving scaled consensus. The Lyapunov-Krasovskii functional is employed to manage the effects of time-varying delays.

MASs with stochastic delay is a sub-category of time-variable delayed MAS. The Consensus control of second-order MAS with stochastic time-varying communication delay has been investigated in [111] and reached the sufficient conditions for system stabilizing by stochastic analysis and Lyapunov stability theory. Also, in [112] a time-varying stochastic nonlinear time-delay MASs has been studied and a quasi-consensus has been proposed to devise a consensus protocol for all agents. In [113], stability analysis of MASs with stochastic delay has been investigated, to achieve this, a general Halanay-type inequality with time-varying coefficients is introduced, leading to a thorough and comprehensive analysis.

A stochastic consensus condition of MASs with both additive time-delays and measurement noises has been considered in [114]. The necessary and sufficient conditions have been obtained by the martingale convergence theorem for stochastic strong consensus. [64] introduced a Stochastic Sampling Mechanism for multi-agent systems (MASs) involving multiple leaders and communication delays. In [Arockia et al., 2022], the study addresses MASs with multiple time-varying delays, establishing the existence and uniqueness of equilibrium points and achieving global asymptotic stability using the Lyapunov-Krasovskii function.

5. Practical overview impact in real work systems

MASs have many applications in control that we investigate some applications and how they affect the delay in their performance.

Cooling systems in ships and power plants that are cooled by water or fluids are one of the applications of MASs, in these systems the paths of the fluid are controlled by control valves to keep the system cool. So, the data temperature of each agent and neighbors and the valve's control command must be transmitted in a network. And any delay in information receiving and transmitting path, even small delay, can damage the system.

The transportation system for materials and products in factories or a cargo handling system at a large airport is another example in MASs. The simplest agent form in these systems consists of at least three one-way conveyors, two switches and controllers and, the required processing. With relation between agents, the materials transfer process is carried out with the required speed and accuracy. With slower path detection, the switches activate other paths to speed up the transfer process. The redirection speed is dependent on the switches' speed and on-time information reception, and delay in information transmission can cause the system to fail.

One of the important applications of MASs is the movement control of mobile robots. These robots are used for surveillance, searching, performing missions in hazardous environments, and so on. Robot control, in order to move in the right direction and avoid collisions between robots, is important in the robot performance, which is achieved by reaching the consensus, and the proper relation between the robots is one of the essential bases of achieving consensus. But the delay in the communication network prevents the consensus reaching. In [79], three robots were considered. Using the formation control method, a robust estimator for delay compensation, the leader state estimation, and its transmission through the communication channel consensus is achieved.

6. Conclusion and future outlook

In this study, we investigate the effect of time-delay in data transfer in control of MASs. In this regard, about 120 papers published since 2013 were studied and two general categories were created based on the location of time-delay and the type of delay. Each of these categories has been divided into more detailed categories, including input delays, output delays, sampling delay, delay in state and feedback in the first category, and fixed delay and time-variable delay in the second category.

In each case, the proposed solution has been considered to solve the delayed problems. Almost, in all categories, network topology plays an important role in reaching consensus, and having a spanning tree can guarantee consensus. Some of the reviewed papers used an estimator to compensate delay. This estimator can predict the delay amount and prevent system instability. Consensus analysis and derived sufficient and necessary conditions have been investigated in mostly researches. Also, the delay effect on industrial applications is another issue that we have addressed in this paper.

As shown that in Fig. 1, researches on delay in MASs has progressed significantly in recent years. Based on reviewed articles, some topics can be considered as future work.

The consensus issue in directed networks with antagonistic interactions and communication delays, as explored in [44], or the output feedback consensus problem for nonlinear multi-agent systems (MASs) with input delays on fixed directed graphs, as examined in [66], can be considered as areas of research in fixed directed networks. Also, for switching graph, the output feedback-based distributed containment control problem for nonlinear MASs with non-identical time-delays in the state under switching directed topologies [102] and the distributed event-triggered tracking consensus [55] and the necessary and sufficient conditions for distributed optimization for containment control [83] are suitable topics. The event-triggered protocol issue, using sampled data for time-varying formations in multi-agent systems (MASs) with multiple leaders, as addressed in [64], also falls under the category of event-triggered topics. Moreover, hybrid MASs with switching communication topology and unreliable communication links [105], fractional-order singular MASs with delay [46], the exponential consensus of the stochastic MAS with uncertain parameters along with the specified leader through [104] are suggested. Future research will explore three key areas: i) advancing cooperative control techniques, including containment, formation, and cluster consensus for second-order multi-agent systems (MASs) with time delays; ii) investigating the impact of additional network-induced constraints, such as packet loss and cyber-attacks, on the consensus of second-order MASs; and iii) extending the proposed methodologies to practical applications, such as aircraft, unmanned marine vehicles, and industrial cyber-physical systems.

As mentioned, other adversaries in MASs are important to system performance. Delay or any disruption to a MAS can be natural or intentional, which in most research has been considered into natural Adversaries. The intentional part of the defects in MASs is located in the cyber-attacks category and consists of noise, disturbance, packet loss, and delays. Each one of these problems can cause the system unstable. Consider this fact that today a large part of the critical infrastructure of any industrial system utilizes network and decentralized systems, attacking these systems can have catastrophic effects. For example, the Stuxnet virus attack on Siemens' physical systems used in many industrial processes in the data monitoring and control network is a catastrophic example of these attacks [116]. Robust control methods have good performance in stochastic systems and are robust against a range of uncertainties, but in physical cyber systems, these uncertainties are unpredictable and attacks are intense and sudden. Therefore, newer control methods are needed to ensure the optimal performance of decentralized systems. The proposed solution is resilient control. This method gives the system the necessary resiliency against cyber-attacks and thus can be used in MASs or any other system that is exposed to cyber-attacks through the use of the data network.

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