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Efficiency Evaluation of Electric Power Distribution: An Analytical Framework using Data Envelopment Analysis

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ABSTRACT

In this study, by using a data envelopment analysis (DEA) model we assess and compare the efficiency of some electricity distribution districts as decision-making units (DMUs) within the Tehran provincial electricity distribution company located in Tehran province, Iran. We termed the model as electric power distribution as the choice of inputs are based on a set of resources that supply and distribute electricity and the outputs will be such as the amount of electrical energy sold and the units of energy delivered. The definition of outputs in most studies is only limited to desirable outputs. However, the outputs in this study consist of both desirable and undesirable variables. Hence, we first extend the conventional DEA model to deal with undesirable outputs. We then provide the efficiency value of each district. Six out of twelve districts were efficient with efficiency score of 1, while others are inefficient with efficiency scores ranged from 0.5594 to 0.9738. The lesser efficiency value can be obtained due to losses in medium voltage transmission lines, low voltage transmission lines and transformers. To address the issues, some solutions such as modifying the network configuration using the existing methods for decreasing power losses and improving the voltage value in distribution network and dividing the power cable into several sections to switch to higher electrical power can be applied.

1. Introduction

Researchers have focused on the electricity distribution efficiency analysis for measuring and comparing the efficiency level of several distribution units. The fundamental aspect of efficiency evaluation is to strive for higher outputs, given the same level of inputs [1]. At the global level, research on sustainability and electricity distribution efficiency aims to economic structure.

The intensification of climate change challenges in Iran has had a significant impact on energy shortages. Over the past decade, constant temperature rises and the significant decrease in rainfalls across Iran have put the country in a hard situation regarding electricity supply during peak consumption periods. Therefore, electricity consumption in Iran has increased by 6,000 megawatts (MW) in the current Iranian year (started on March 20, 2024) compared to the previous year (Tehran Times, 2024), reported by the Head of Iran's Power Generation, Distribution, and Transmission Company (known as Tavanir). Tavanir has repeatedly announced that the company is implementing a variety of programmes for managing the situation and preventing blackouts in the country. Iran's economy is characterized by its hydrocarbon, agricultural, and service sectors, in addition to manufacturing and financial services, with over 40 industries directly involved in the Tehran province (Tehran Times, 2024). In Iran, mainly state-owned power companies control the distribution, transmission and

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generation of electricity. For these companies, generating electricity is the first step in delivering electricity and the next steps are transmission and distribution. After generating electricity in power plants, the process of electrical energy transmission from the generator or producer to the distribution station near cities or industrial centres is carried out. Next, it is the distribution and delivery of electrical energy to consumers. The main issues in electrical power distribution are quality, continuity and power. Disruptions in production units will cause interruptions in energy supply and consequences for businesses and electricity customers [2]. Electricity distribution networks are in direct contact with electricity consumers and the problems of distribution systems in this industry will be considered as a problem of the entire electricity industry by consumers. Hence, continuous development of electrical energy distribution system is necessary and inevitable.

The performance of a company can be achieved by comparing the company's performance with other utility companies that perform similar functions. In this case, the electricity distribution efficiency can be provided for measuring and comparing the efficiency level of several distribution units. Data envelopment analysis (DEA) model as a performance evaluation technique was proposed by Charnes et al. [3] for developing best practices and benchmarking to assess the overall efficiency of decision-making units (DMUs). Although DEA provides a readily available framework, it is not so straight forward as outputs in environmental efficiency models make up both desirable and undesirable outputs. For instance, higher GDP index tend to come with higher energy-related carbon dioxide (CO_2) emissions. This means that desirable outputs have to be sacrificed so that inputs can be reallocated for minimization of undesirable outputs [4].

Despite the problems noted, there is a large number of DEA applications in environmental performance, especially at the national level (see [5]). Färe et al. [6] first proposed an environmental assessment model based on a nonparametric DEA framework, which considered both desirable and undesirable outputs together. Since then, many researchers began to provide a variation of one of the following models and measures: energy intensity and even those that account for linguistic preferences (see [2,7,8]). DEA has been further used to assess performance in the four responsibilities of each energy sector consisting of production, transmission, distribution, and retail [2]. Medeiros et al. [9] also used some DEA methods such as cross-efficiency and the ratio-based efficiency techniques to provide the efficiency of and rank some electric distribution companies. The definition of output variables in recent studies is only limited to desirable outputs. However, in our study, the output variables consist of both desirable and undesirable variables. In this case, we would be able to employ a more variety of variables to provide more reasonable results.

A few studies have incorporated undesirable outputs in electricity distribution efficiency. For instance, Khalili-Damghani et al. [10] developed a network data envelopment analysis model to evaluate the efficiency of electric power production and distribution processes. The emissions released from the production process are considered as an undesirable output in their study. As the data are mixed with considerable amount of uncertainty their proposed approach is developed using interval data. Tavassoli et al. [11] further proposed a network data envelopment analysis model for assessing the sustainability of Iran's Electricity Distribution Networks and their components. Their proposed method has the following characteristics: All generation, transmission, and distribution stages, as well as the overall performance of the network, are evaluated in a unified framework; undesirable outputs, re-work, and external inputs are considered in the model. Their proposed method deals with undesirable outputs. However, Iran's electricity distribution networks are determined as DMUs in their studies and the data of each DMU collected from all over the Iran country with different climate conditions. In this case, the DMUs are not exactly homogeneous and they can be considered as heterogeneous DMUs.

In this study, by using a DEA model we assess and compare the efficiency of electricity distribution districts within the Tehran provincial electricity distribution company located in southern, southeastern, and southwestern Tehran province, Iran. These twelve districts of Tehran province have almost the same climate. Therefore, their performance can be evaluated while they have almost the same climate conditions.

1.1. DEA

Data envelopment analysis (DEA) was first proposed by Charnes et al. [3] for providing the technical efficiency of a set of homogenous DMUs based on their respective multiple inputs and outputs. Schools, hospitals, banks, governments, national economies, and economic sectors are examples of DMUs. The inputs can consist of labour, materials, energy, machines, and other resources, while the by-product of outputs may consist of finished products, services, customer satisfaction, and other forms of outcomes. The conventional DEA model (see Appendix A) is usually able to provide the efficiency values of DMUs based on the assumption that inputs have to be minimized and outputs have to be maximized as desirable (good) input-output variables. For instance, if we consider investment as a single input and products as single output for DMUs. In this case, in order to improve the performance of an inefficient DMU, investment as desirable input should be decreased while products as desirable output should be increased. However, in real world example and certain cases, some input/output data may be presented as undesirable (bad) inputs/outputs in the production process. In this case, for improving the performance of an inefficient DMU, the undesirable outputs and desirable inputs should be decreased while the desirable outputs and undesirable inputs should be increased [12]. For example, by considering the number of accidents as output in a case of road safety assessment, in order to improve the performance of an inefficient DMU (Road), the number of accidents as undesirable output should be decreased. Hence, the classical DEA model should be extended to deal with such inputs or/and outputs.

In this study the convectional DEA model would be extended to deal with undesirable outputs. The twelve electricity distribution districts of Tehran province with almost the same climate were determined as DMUs. Therefore, their performance can be evaluated while they have almost the same climate conditions as homogeneous DMUs.

The remainder of the paper is organized as follows: Section 2 explains the methodology. Section 3 presents data collection. Section 4 discusses results and Section 5 concludes the study.

2. Methodology

Suppose there are n DMUs to be evaluated, which use m inputs (x_{ij}, $i = 1, \dots, m$, $j = 1, \dots, n$) to produce s outputs (y_{rj}, $r = 1, \dots, s$, $j = 1, \dots, n$).

2.1. The envelopment input-oriented form of DEA

The envelopment input-oriented form of DEA model [3] as the dual form of the DEA model (A.3) in Appendix A can be formulated as the following linear programming (LP) problem.

$$\begin{aligned} & \min \theta_o \\ s.t.\ & \textstyle\sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_o x_{io}, \quad i = 1, \dots, m, \\ & \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} \ge y_{ro}, \quad r = 1, \dots, s, \\ & \lambda_j \ge 0, \quad j = 1, \dots, n, \end{aligned} \qquad (1)$$

where $\lambda_1, \dots, \lambda_n$ are non-negative variables. DMU$_o$ (the DMU under evaluation) is efficient if the optimal value of the objective function (θ_0^*) in the above LP problem is equal to 1, and is considered inefficient if $\theta_0^* < 1$.

2.2. The envelopment output-oriented form of DEA

Similar to model (1), the envelopment output-oriented form of DEA model can be expressed as

$$\begin{aligned} & \max \varphi_o \\ s.t.\ & \textstyle\sum_{j=1}^{n} \lambda_j x_{ij} \le x_{io}, \quad i = 1, \dots, m, \\ & \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} \ge \varphi_o y_{ro}, \quad r = 1, \dots, s, \\ & \lambda_j \ge 0, \quad j = 1, \dots, n, \end{aligned} \qquad (2)$$

where λ_j is defined as in the model (1). DMU$_o$ is efficient if the optimal value of the objective function (φ_o^*) in the above LP problem (2) is equal to 1, and is considered inefficient if $\varphi_o^* > 1$. Note that $\varphi_o^* \ge 1$, and $\varphi_o^* = 1$ in the above model (2) if and only if $\theta_o^* = 1$ in model (1). This indicates that models (1) and (2) identify the same frontier, in which $\theta_o^* = {}^{1}/_{\varphi_o^*}$.

2.3. The DEA model with desirable inputs and both desirable and undesirable outputs

By considering DEA models (1) and (2), the DEA model with desirable inputs and both desirable and undesirable outputs can be formulated as follows [13]:

$$\begin{aligned} & \min \psi_0 = \frac{\theta_o}{\varphi_o} \\ s.t.\ & \textstyle\sum_{j=1}^{n} \lambda_j x_{ij} \le \theta_o x_{io}, \quad i = 1, \dots, m, \\ & \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} \ge \varphi_o y_{ro}, \quad r \in R^D, \\ & \textstyle\sum_{j=1}^{n} \lambda_j y_{rj} \le \theta_o y_{ro}, \quad r \in R^U, \\ & 0 < \theta_o \le 1, \ \varphi_o \ge 1, \\ & \lambda_j \ge 0, \quad j = 1, \dots, n, \end{aligned} \qquad (3)$$

where λ_j is defined as in the model (1). The efficiency score of DMU$_o$ is equal to the optimal value of $\psi_0 = \frac{\theta_o}{\varphi_o}$. DMU$_o$ is efficient if the optimal value of ψ_0 in the above model (3) is equal to 1 (i.e., $\theta_o^* = \varphi_o^* = 1$). Otherwise, DMU$_o$ is inefficient. It was assumed that the outputs can be partitioned into the subsets of desirable (D) and undesirable (U), in which $R = R^D \cup R^U$, $R^D \cap R^U = \emptyset$, $R = \{1, \dots, s\}$ and s is defined as in the models (1) and (2). It should be mentioned that the index sets R^D and R^U are associated with desirable and undesirable outputs.

A list of notations and indexes regarding to the above-mentioned LP problems provided and listed as follows:

Notations & Indexes	**Description**
θ_o	The objective function of DEA model (1) in which, the optimal value of θ_o is the efficiency value of DMU$_o$ for input-oriented form
λ_j	The non-negative variable associated with DMU$_j$
φ_o	The objective function of DEA model (2) in which, the optimal value of φ_o is the efficiency value of DMU$_o$ for output-oriented form
ψ_0	The objective function of DEA model (3) in which, the optimal value of ψ_0 is the efficiency value of DMU$_o$ for extended DEA form
R	The index set of outputs
R^D	The index set assigned to desirable outputs
R^U	The index set assigned to undesirable outputs

The following flowchart is provided to illustrate the DEA process and the extended model clearly (see Fig. 1).

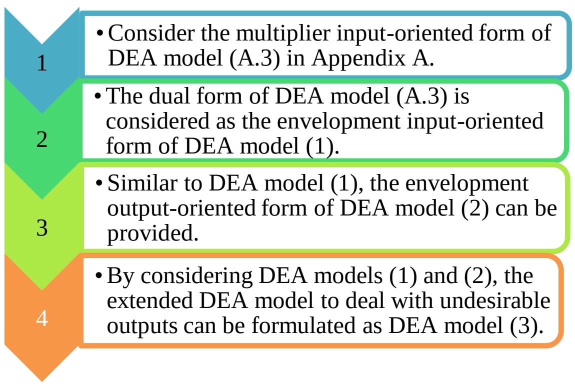

Fig. 1. Illustration of DEA process and the extended DEA model.

3. Data Collection

In this section we determine the input and output variables that are used in evaluating the relative efficiency of the selected DMUs. In DEA, the technical efficiency of a set of homogenous DMUs can be provided based on their respective multiple inputs and outputs. Normally, any resource used by a DMU should be treated as an input variable, and the output variables come from the performance and activity measures that result when a DMU converts resources to produce products or services [14].

Power distribution is key for efficient energy management. Properly designed and maintained power distribution systems ensure a reliable supply of electricity. This reliability is essential and critical for homes, hospitals, industries, companies and other sectors that depend on continuous electricity.

This research was conducted in order to provide and compare the efficiency of several electricity distribution districts within the Tehran provincial electricity distribution company located in southern, southeastern, and southwestern Tehran province, Iran. The twelve districts or cities belong to Tehran provincial electricity distribution company, namely Pakdasht, Pishva, Varamin, Qarchak, Rey City, Islamshar, Golestan, Boustan, Robat Karim, Kahrizak, Khavaran, and Chahar Dangeh. These districts of Tehran province have almost the same climate. Therefore, their performance can be evaluated while they have almost the same climate conditions. Thus, each district can be considered as a DMU and the efficiency level of these DMUs would be evaluated in this study. We termed the model as electric power distribution as the choice of inputs are based on a set of resources that supply and distribute electricity and the outputs will be such as the amount of electricity sold and the units of energy delivered. The operational definition of the 6 inputs and 4 outputs are as following way.

Input Variables:

x_1: The medium voltage transmission lines to number of distribution substation ratio (Meters/Devices)

x_2: The low voltage transmission lines to number of distribution substation ratio (Meters/Devices)

x_3: Average power of air substation (KVA/Devices)

x_4: Average power of ground substation (KVA/Devices)

x_5: The amount of electricity purchased (Million Rial)

x_6: Number of existing air and ground substation

Output Variables:

y_1: The amount of undistributed electricity (MWH)

y_2: The number of power outage of medium voltage to medium voltage transmission lines ratio (Number/KM)

y_3: The number of power outage of low voltage to low voltage transmission lines ratio (Number/KM)

y_4: The amount of electrical energy sold (Million Rial)

With respect to output 1 (y_1), in certain cases, because of the incorrect forecasting of required load, predicted and unpredicted losses of distribution lines, some amount of the electricity purchased cannot be distributed which is defined as the amount of undistributed electricity. In this case, for improving the performance of an inefficient DMU, output 1 a long with outputs 2 and 3 (i.e., y_1, y_2, & y_3) should be decreased. Therefore, outputs 1, 2 and 3 (i.e., y_1, y_2, & y_3) are considered as undesirable outputs and output 4 (i.e., y_4) is a desirable output. The 6-input and 4-output dataset of these twelve districts presented in Table 1.

It should be mentioned that the dataset is real and collected from the website of Electric Energy Distribution Company of Tehran Province (i.e., https://www.tvedc.ir) for the period of a year (i.e., 21/Mar/2022 – 20/Mar/2023).

4. Results and Discussion

By using DEA model (3) and employing the dataset in Table 1, the results of efficiency value for each district are provided and presented in Table 2 and Fig. 2. The Wolfram Mathematica software is used to obtain and provide the results of efficiency values. The Wolfram Language has a collection of algorithms for solving linear optimization problems with real variables. The results can be interpreted as following way. From Table 2 and Fig. 2, it can be mentioned that six DMUs (districts) are efficient among the twelve districts (i.e. the efficiency score of districts Rey City, Islamshahr, Golestan, Kahrizak, Khavaran, and Chahr Dangeh is equal to 1, so these districts are efficient). It means all the resource usage of these districts reached an optimal status for the combination of input factors and production scale. While, the efficiency values of districts Pakdasht, Pishva, Varamin, Qarchak, Boustan, and Robat Karim are equal to 0.7280, 0.5594, 0.9738, 8785, 0.8229, and 0.7727 respectively, so they are inefficient (see Table 2).

The performance of inefficient DMUs in a classical DEA model can be improved by reducing the input and/or increasing the output. By considering the objective function of DEA model (A.1) in Appendix A in which, the efficiency value of a DMU is equal to $\frac{The\ weighted\ sum\ of\ ouputs}{The\ weighted\ sum\ of\ inputs}$, it would be clear, reducing the inputs or increasing the outputs can improve the efficiency and performance of an inefficient district. When there exit undesirable outputs, the undesirable outputs can be treated as normal inputs (see [15]).

By considering the input-output data and results provided, it can be mentioned that lower efficiency occurs due to losses in medium voltage transmission lines, low voltage transmission lines and transformers. These issues would be appeared due to the electrical cable being too lengthy or not in compliance with the standard or losses in the transformer and low-capacity usage. Moreover, the amount of undistributed electricity (due to lack of managing of proper and optimal electricity distribution) can be mentioned as another reason for the lower efficiency. To deal with the issues some ideas such as modifying the network configuration by dividing the power cable into several sections and improving the electricity distribution management for optimal electricity distribution can be proposed.

If one were to compare the results of efficient and inefficient districts, it can be concluded that the resource usage of inefficient districts did not reach the optimal status for the combination of input factors and production scale. It would be possible that the inefficient districts share common characteristics such as losses in voltage transmission lines, losses in the transformer or higher power losses. Therefore, the inefficiency occurs due to losses in voltage transmission lines and transformers as the main reasons. However, the amount of undistributed electricity can be mentioned as another reason of inefficiency. It can be noted that losses in voltage transmission lines and transformers is up to the 80 percentage of inefficiency reason.

Table 1. The dataset of the twelve districts

DMU (or District)	Inputs						Outputs			
	x_1	x_2	x_3	x_4	x_5	x_6	y_1	y_2	y_3	y_4
Pakdasht	211	174	183	760	2833093	5471	2124	1.72	4.14	3029751
Pishva	312	254	123	557	523405	1575	407	1.06	3.34	514145
Varamin	308	134	150	773	1666063	3983	1192	0.84	6.61	1998673
Qarchak	204	231	165	830	971254	2411	504	1.96	4.90	1425254
Rey City	329	716	215	810	1538879	1600	463	2.87	2.45	2603084
Islamshahr	232	1110	181	837	1372104	2029	593	2.17	0.98	1977076
Golestan	132	206	184	769	863723	1891	257	3.20	4.87	1341921
Boustan	192	425	178	783	532177	908	195	2.63	3.41	758005
Robat Karim	180	172	144	788	1208603	3579	1111	1.67	2.54	1601942
Kahrizak	200	126	188	823	3067124	6089	1285	1.89	2.44	4617774
Khavarn	198	506	204	874	479445	1561	251	2.03	1.10	874944
Chahr Dangeh	169	160	151	726	1359848	3495	1056	1.68	3.04	2354382

Note: The data are taken from https://www.tvedc.ir/#auction1219 (21/Mar/2022 – 20/Mar/2023)

Table 2. The results of efficiency based on the dataset in Table 1

DMU (or District)	Efficiency value
Pakdasht	0.7280
Pishva	0.5594
Varamin	0.9738
Qarchak	0.8785
Rey City	1
Islamshahr	1
Golestan	1
Boustan	0.8229
Robat Karim	0.7727
Kahrizak	1
Khavaran	1
Chahr Dangeh	1

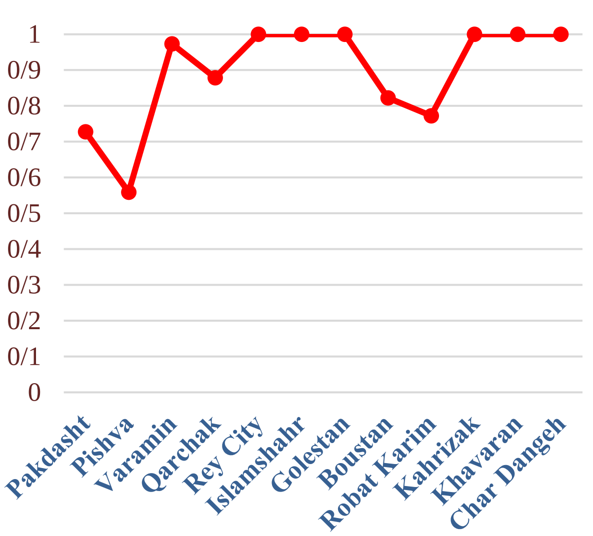

Fig. 2. Results of efficiency

As a cost-benefit analysis, it can be noted that the costs consist of resources and capital that must be spent on an activity or project such as financial costs (purchase of equipment, salaries and wages, raw materials), time costs (time that must be spent on a project), social or environmental costs (possible damage to the environment or society). Benefits also refer to the values and are obtained from carrying out an activity or project. The benefits can be tangible or intangible such as financial benefit (income, increased sales), improved quality of life or productivity, positive social or environmental impacts. In DEA costs are analogous to inputs and benefits are analogous to outputs. DEA considers the positive economic and social impacts of the project and provides a comprehensive assessment of the industrial units. By considering the efficiency results provided and listed in Table 2 and the scenario of cost-benefit of solutions, it can be concluded that the efficient districts indicate the desirability of a policy, an economic plan, or an investment plan. While, the inefficient districts indicate undesirability of a policy or an economic plan.

In comparison with existing and recent studies on the efficiency analysis of electric distribution companies using DEA, we did not use the conventional DEA model to provide the efficiency value of each DMU. In a real-world example and certain cases, some input/output data may be presented as undesirable (bad) inputs/outputs in the production process. For instance, in our case study, the output variables consist of both desirable and undesirable variables. Hence, we first extended the conventional DEA model to be able to provide the efficiency level of those DMUs (electricity distribution districts) based on the assumption of desirable input and both desirable and undesirable output variables. In this case, we would be able to employ a more variety of variables to provide more reasonable results.

The advantage of our study against the existing and recent studies is the ability of the proposed extended DEA model which is able to achieve the efficiency score of those electricity distribution districts based on the assumption of desirable and undesirable outputs. Furthermore, the twelve electricity distribution districts with almost the same climate were determined as DMUs in this study. Thus, the performance of each district can be evaluated while they have almost the same climate conditions as homogeneous DMUs. It can be concluded that the results provided in our study can be more trusted than those provided by others.

5. Conclusion and Remarks

In this study we evaluated the efficiency level of some electricity distribution districts with almost the same climate conditions as DMUs within the Tehran provincial electricity distribution company located in Tehran province, Iran by using a DEA model. We found that among the twelve districts, there are six districts (DMUs) as efficient and the rest of districts are inefficient. On the other hand, the performance of the 6 districts as the efficient DMUs in comparison with the rest 6 inefficient districts is the best. The lower efficiency values occur due to losses in medium voltage transmission lines, low voltage transmission lines and transformers. The inefficient districts share common characteristics such as losses in voltage transmission lines, losses in the transformer or higher power losses. These issues would be appeared due to the electrical cable being too lengthy or not in compliance with the standard or losses in the transformer and low-capacity usage. The amount of undistributed electricity due to the lack of electricity distribution management of correct and balanced is another reason for the lower efficiency value. To deal with the issues, modifying the network configuration by using the existing methods for decreasing power losses and dividing the power cable into several sections as well as improving the electricity distribution management for optimal distribution of available electricity and using small electrical power to switch to higher electrical power can be proposed as solutions. Moreover, to address inefficiencies in medium-voltage and low-voltage lines, the network reconfiguration in districts with higher number of power outage should be prioritized.

The static nature of the DEA model as another issue and the data availability restricted to a single year or region as another limitation does not account for dynamic changes over time. However, a relational DEA model can be improved to measure the efficiency of a dynamic system as a future direction. Therefore, as a suggestion of future research, a relational DEA model can be improved to assess efficiency over multiple years. Additional variables such as maintenance costs or energy quality can be further recommended as a future direction.

Appendix A.

A common value of relative efficiency when there are multiple inputs and outputs can be expressed as

$$\frac{The\ weighted\ sum\ of\ ouputs}{The\ weighted\ sum\ of\ inputs}.$$

By using this notion, the efficiency measurement is generalized for a set of homogenous DMUs from a single-output and single-input to multiple-outputs and multiple-inputs. The DMU under evaluation (the target DMU) is known as DMU_o where o ranges over $1, 2, \dots, n$.

DEA model:

Consider the relative efficiency of n DMUs which use m inputs ($x_{ij}, i = 1, \dots, m,\ j = 1, \dots, n$) to produce s outputs ($y_{rj}, r = 1, \dots, s,\ j = 1, \dots, n$). By assuming that the inputs-outputs data are nonnegative and at least one input and one output are positive, the following fractional programming problem is solved for each DMU to obtain measures of the input weights ($v_i,\ i = 1, \dots, m$) and the output weights ($u_r,\ r = 1, \dots, s$) as variables.

$$\max \theta_o = \frac{u_1 y_{1o} + u_2 y_{2o} + \cdots + u_s y_{so}}{v_1 x_{1o} + v_2 x_{2o} + \cdots + v_m x_{mo}}$$

subject to:

$$\frac{u_1 y_{1j} + u_2 y_{2j} + \cdots + u_s y_{sj}}{v_1 x_{1j} + v_2 x_{2j} + \cdots + v_m x_{mj}} \le 1, \quad j = 1, \dots, n,$$
$$u_1, u_2, \dots, u_s \ge 0,$$
$$v_1, v_2, \dots, v_m \ge 0, \qquad \text{(A.1)}$$

where

y_{rj} = amount of output r associated with DMU_j

u_r = weight associated with output r

x_{ij} = amount of input i associated with DMU_j

v_i = weight associated with intput i.

In DEA model (A.1), we use the optimal value of the objective function to evaluate the efficiency value of DMU_o, which is equal to

$$\theta_o^* = \frac{u_1^* y_{1o} + u_2^* y_{2o} + \cdots + u_s^* y_{so}}{v_1^* x_{1o} + v_2^* x_{2o} + \cdots + v_m^* x_{mo}}.$$

According to the transformation approach proposed by Charnes & Cooper [16], a “linear fractional programming problem” can be modified into an equivalent linear programming problem, thus the above model (A.1) can be replaced by the following linear programming problem,

$$\max \theta_o = u_1 y_{1o} + u_2 y_{2o} + \cdots + u_s y_{so}$$

subject to:

$$v_1 x_{1o} + v_2 x_{2o} + \cdots + v_m x_{mo} = 1,$$
$$u_1 y_{1j} + u_2 y_{2j} + \cdots + u_s y_{sj} \le$$
$$v_1 x_{1j} + v_2 x_{2j} + \cdots + v_m x_{mj}, \quad j = 1, \dots, n,$$
$$u_1, u_2, \dots, u_s \ge 0,$$
$$v_1, v_2, \dots, v_m \ge 0. \qquad \text{(A.2)}$$

We note that the values of efficiency are independent of the units, in which the inputs-outputs are measured, thus establishing these units to be the same for every DMU. The above DEA model as the multiplier and input-oriented can be written as follows:

$$\max \theta_o = \sum_{r=1}^{s} u_r y_{ro}$$

subject to:

$$\sum_{r=1}^{s} v_i x_{io} = 1,$$
$$\sum_{r=1}^{s} u_r y_{rj} - \sum_{r=1}^{s} v_i x_{ij} \le 0, \quad j = 1, \dots, n,$$
$$u_1, u_2, \dots, u_s \ge 0,$$
$$v_1, v_2, \dots, v_m \ge 0. \qquad \text{(A.3)}$$

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