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Adaptive Fault Tolerant Consensus Tracking Control based on Cubic Observers for Lipschitz Nonlinear Multi-Agent Systems

Mahsa Hasanshahi [1] , Malihe Maghfoori Farsangi [1,*] , Elham Amini Boroujeni [2]

[1] *Department of Electrical Engineering, Shahid Bahonar University of Kerman, Kerman, Iran*
[2] *Department of Electrical and Computer Engineering Faculty , kharazmi University, Tehran, Iran*

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ABSTRACT

This paper presents a fault-tolerant consensus tracking approach that integrates cubic observers This paper presents a fault-tolerant consensus tracking approach that integrates cubic observers with adaptive control to enhance the performance of Lipschitz nonlinear multi-agent systems (MASs) subject to simultaneous actuator and sensor faults. The proposed method enables precise real-time fault estimation and state estimation, ensuring accurate system operation. A distributed adaptive control strategy, informed by the cubic observers, dynamically compensates for actuator and sensor faults in multi agent systems, preserving stability and consensus among agents. The controller parameters are systematically derived using Linear Matrix Inequalities (LMIs), providing a rigorous theoretical foundation. Under the above objectives, Lyapunov-based stability analysis guarantees that the tracking errors remain uniformly ultimately bounded and converge over time. Simulation examples validate the effectiveness of the proposed approach, demonstrating consistent performance under simultaneous and time-varying faults. Notably, the Root Mean Square Error (RMSE) analysis confirms that our method performance compared to existing approaches, significantly reducing tracking errors in adverse conditions.

1. Introduction

Over the past few decades, multi-agent systems (MASs) have seen remarkable advancements, mainly due to their ability to handle complex tasks beyond the capabilities of individual agents. These systems are extensively applicable in various fields such as mobile sensor networks, unmanned aerial vehicles, and satellite clusters[1]. However, many real-world MASs exhibit nonlinear or strongly coupled dynamics, making linear models inadequate. Relying on linear approximations often leads to significant modeling errors and overly conservative designs. In response to these challenges, recent research has increasingly focused on Lipschitz nonlinear dynamics, which offer a broader and more flexible modeling framework. Since any smooth nonlinear function can be transformed into a Lipschitz form, these systems have gained significant attention. Several approaches have been developed to address control and consensus in such systems, including continuous-discrete time observer-based protocols [2], adaptive observers for sensor faults [3], and sliding-mode strategies under switching topologies [4]. Fully distributed adaptive fault-tolerant consensus protocols for Lipschitz nonlinear MASs with actuator faults were

* Corresponding author
E-mail address: mmaghfoori@uk.ac.ir
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proposed in [5]. Despite these advances, existing methods primarily focus on either consensus control or fault detection but rarely integrate both aspects in a unified framework.

The increasing complexity of engineering systems, driven by technological advancements, has led to a higher risk of faults in multi-agent systems (MASs) compared to single-agent systems. This is due to the greater number of actuators, sensors, and interconnected components involved. Fault-tolerant control (FTC) techniques have been developed to ensure stable operation despite system failures[6].Various strategies have been developed to maintain consensus despite faults. These include using virtual actuators and sensors to reconfigure faulty systems[7], adaptive estimation of fault parameters[8], and optimization-based algorithms[9]. Techniques like sliding mode control[4] and Lyapunov-based approaches[7, 8], are commonly used to ensure stability and robustness.

Addressing actuator faults specifically, [10] develops protocols for linear MASs, while event-triggered strategies [11] and robust H_∞-based control [12] have been explored for nonlinear MASs. Further efforts, such as[13], propose a fault-tolerant control method for nonlinear multi-agent systems facing output constraints addressing actuator failures.

However, a critical limitation in the existing literature is the lack of comprehensive strategies for handling simultaneous actuator and sensor faults in MASs. Most studies focus exclusively on actuator faults, overlooking the significant impact of sensor faults on overall system performance. Since neighboring agents rely on shared information, sensor faults can propagate incorrect measurements across the entire MAS, causing severe degradation in control and consensus performance. Addressing this challenge requires an integrated fault-tolerant strategy that accounts for both actuator and sensor failures. Several attempts have been made in this direction, such as adaptive descriptor observer-based schemes [14], finite-time sliding mode-based observers [15], and distributed adaptive FTC strategy [16]. Additionally, The fault-tolerant formation control problem of a class of nonlinear multiagent systems with actuator/sensor faults under directed and switching network topology is addressed in [17].

Moreover, in many practical systems, states information is often unavailable, making consensus control protocols that rely on full state access inapplicable. To address this challenge, observer-based techniques have been introduced as a potential solution. Reference [18] introduces a distributed fault estimation observer for actuator faults, and [3] transforms sensor faults into equivalent actuator faults and designing a distributed adaptive fault estimation observer. Further advancing observer-based strategies, [19] presents a distributed-observer-based adaptive control scheme for nonlinear MASs with sensor faults without employing fault estimation strategies. A complementary approach is introduced in [20], which combines fault estimation with fault-tolerant tracking control, proposing a sliding-mode observer-based method for linear MASs affected by actuator faults. To facilitate comparison with other works in the literature review, we have included Table1.

To address the challenges of simultaneous actuator and sensor faults in Multi-Agent Systems (MAS), this paper proposes a distributed adaptive observer-based fault-tolerant consensus control strategy. The approach utilizes cubic observers [21], [27], which offer superior convergence rates and enhanced robustness compared to traditional linear or sliding-mode designs. While these cubic-based frameworks have been applied to fault-tolerant control in linear systems [25] and leader-follower linear MAS [26], the current work extends these methodologies to nonlinear Lipschitz-type MAS. This extension is necessary in nonlinear MAS, where agent interactions and communication must be managed in the presence of simultaneous sensor and actuator faults. The proposed strategy integrates fault estimation with fault-tolerant control mechanisms to detect and compensate for faults while maintaining system integrity during dynamic interactions. By leveraging the properties of cubic observers, the proposed approach addresses fault tolerance against concurrent actuator and sensor failures. The main contributions are:

1. **Simultaneous Fault Handling:** We address the simultaneous occurrence of both actuator and sensor faults, a scenario that distinguishes this work from studies focusing on isolated fault types [3, 20] or individual fault categories [14, 15, 16]. This dual consideration significantly broadens the applicability of our findings to real-world engineering scenarios where both types of faults are likely to occur, enhancing system resilience.
2. **Integration of Fault Estimation:** Unlike works in [12] and [19] that overlook explicit fault estimation, our strategy incorporates a distributed adaptive fault estimation mechanism for Lipschitz nonlinear multi-agent systems.
3. **Dynamic Adaptive Control Mechanism:** We develop an adaptive control mechanism that leverages cubic observers to establish a tracking control strategy, ensuring stable operation and dependable performance of the multi-agent system under adverse conditions.
4. **Enhanced Fault Tolerance:** The proposed method improves fault tolerance in the face of simultaneous faults, providing a reliable framework for complex, interconnected MASs.

The structure of the paper is as follows: Section II covers the necessary preliminaries and formulates the problem. Section III presents the main results. In Section IV, two examples are provided, and Section V offers concluding remarks.

Table 1. of Fault-Tolerant Consensus Approaches in MASs Comparison

Reference	*System Dynamics*	*Fault Type Addressed*	*Control Strategy*	*Fault Estimation*	*Observer Type*
[5]	*Lipschitz Nonlinear*	*Actuator faults*	*Fully distributed adaptive control*	*No*	*No observer*
[10]	*Linear*	*Actuator faults*	*Fully distributed adaptive control*	*No*	*No observer*
[11]	*Linear*	*Actuator faults*	*Event-triggered adaptive control*	*No*	*No observer*
[12]	*Lipschitz Nonlinear*	*Actuator faults*	*Distributed robust H∞ control*	*No*	*No observer*
[13]	*Nonlinear*	*Actuator faults*	*Adaptive Radial basis function neural networks and backstepping control*	*No*	*No observer*
[18]	*Lipschitz Nonlinear*	*Actuator faults*	*Fully distributed adaptive control*	*Yes*	*Distributed output estimation error observer*
[3]	*Lipschitz Nonlinear*	*Sensor faults*	*Robust fault tolerant control*	*Yes*	*Distributed adaptive observer*
[19]	*Nonlinear*	*Sensor faults*	*Distributed adaptive observer-based control*	*No*	*Distributed adaptive observer*
[14]	*Nonlinear*	*Actuator & Sensor faults*	*Formation containment control*	*Yes*	*Adaptive descriptor observer*
[15]	*Linear*	*Actuator & Sensor faults*	*Descriptor approach-based distributed robust FTC*	*Yes*	*Finite-time sliding mode observer*
[16]	*Linear*	*Actuator & Sensor faults*	*Distributed adaptive control*	*Yes*	*Sliding mode-based unknown input observer*
[17]	*Nonlinear*	*Actuator & Sensor faults*	*formation fuzzy FTC*	*No*	*Leuenberger observer*
[20]	*Linear*	*Actuator & Sensor faults*	*Distributed fault tolerant control*	*Yes*	*Dynamic output feedback observer*
Proposed Approach	*Lipschitz Nonlinear*	*Actuator & Sensor faults*	*Distributed Adaptive fault tolerant control with fault compensation*	*Yes*	*Cubic observer*

1. Problem Statement:

1.1. Graph theory and notations

In multi-agent systems, consider each agent as a node, with the information exchange among N agents represented by a directed graph $\mathcal{G} = (V, \varepsilon, \mathcal{A})$ where $V = \{v_1, v_2, \dots, v_N\}$ is the set of nodes (agents), $\varepsilon \subset V \times V$ is the set of edges $\mathcal{A} = a_{ij} \in R^{N\times N}$ is the adjacency matrix. If $(v_j, v_i) \in \varepsilon$, it indicates that node j is a neighbor of node i, or in other words, node i can receive information from node j. The adjacency matrix $\mathcal{A}$ is defined as

$$\mathcal{A} = \begin{cases} a_{ij} = 1 & (v_j, v_i) \in \varepsilon \\ a_{ij} = 0 & (v_j, v_i) \notin \varepsilon \end{cases}$$

where a_{ij} is the element of the adjacency matrix located in row i and column j. The Laplacian matrix is defined as $\mathcal{L} = l_{ij} \in R^{N\times N}$, where $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}, i \neq j$. The graph $\bar{\mathcal{G}}$ is introduced to represent the communication topology between the leader node and its neighbors. For the graph $\bar{\mathcal{G}}$, the matrix $G = diag(g_1, g_2, \dots, g_N)$ is defined to represent that there is a directed path form node i to every other node. The scalar $g_i = 1$, if there is a connection between the node i and the leader node; otherwise, $g_i = 0$.

Notation: In this paper, I_N denotes the identity matrix of size $N \times N$, $R^{m\times n}$ represents a real matrix of dimensions $m \times n$, $\otimes$ denotes the Kronecker product, and $diag(a_1, a_2, \dots, a_N)$ represents a diagonal matrix with elements $a_1, a_2, \dots, a_N$. The symbol $\|.\|$ denotes the matrix norm. For a symmetric matrix A, the notation $A >$

0 indicates that the matrix is positive definite, meaning that $x^T A x > 0 \ \forall x \in R^n$.

1.2. System description

Consider a multi-agent system (MAS) comprising a leader and M followers, labeled as 0 (leader) and $1, \dots, M$ (followers). The dynamic of the leader and followers are described as follows:

$$\begin{aligned} \dot{x}_0(t) &= A x_0(t) + f_L(x_0(t), t) \\ \dot{x}_i(t) &= A x_i(t) + B(u_i(t) + \delta_i^a) + f_L(x_i(t), t) \\ y_i(t) &= C x_i(t) + \delta_i^s \end{aligned} \quad (1)$$

where $x_0(t)$ and $f_L(x_0)$ are the state vector and the nonlinear function of leader. $x_i(t) \in \mathbb{R}^n$, $u_i(t) \in \mathbb{R}^m$ *and* $y_i(t) \in \mathbb{R}^p$ represent the state, input, output vectors of followers, respectively. Matrices A, B and C are known and have appropriate dimensions. The terms δ_i^a and δ_i^s denote the actuator and sensor faults, respectively. The nonlinear function $f_L(x_i(t), t)$ is assumed to be Lipschitz in x with a Lipschitz constant $\gamma > 0$, i.e.,

$$\|f_L(x_1) - f_L(x_2)\| \le \gamma \|x_1 - x_2\|, \qquad \forall x_1, x_2 \in \mathbb{R}^n$$

The following basic assumption are required to facilitate the analysis:

Assumption 1. The unknown actuator and sensor faults and their time derivatives are bounded, i.e., There exist the constants A_1, A_2 and S_1, S_2 such that $\|\delta^a(t)\| \le A_1$, $\|\dot{\delta}^a(t)\| \le A_2$ *and* $\|\delta^s(t)\| \le S_1$, $\|\dot{\delta}^s(t)\| \le S_2$.

To evaluate the proposed protocol in this paper, we present the following stability definition and related lemmas.

Definition 1. The signal $z(t)$ is considered to be uniformly ultimately bounded (UUB) with the ultimate bound b, if for positive constants b and c and for every $a \in (0, c)$, there exists $T(a, b)$ independent of t_0, such that $\|z(t)\| \le a$ implies $\|z(t)\| \le b$ for all $t \ge t_0 + T$. [22]

Lemma 1. Given a positive constant ε, the following inequality holds

$$2x^T y \le \varepsilon x^T x + \varepsilon^{-1} y^T y, \qquad x, y \in \mathbb{R}^n$$

Lemma 2. Schur Complements[23]: For the matrices X, Y, and W, with appropriate dimensions, where $X^T = X$, $Y^T = Y$, then the following inequalities hold true in an equivalent manner.

1. $\begin{bmatrix} X & W \\ W^T & Y \end{bmatrix} < 0$

2. Each of the following is valid

(ı) $Y < 0$, *and* $X - W Y^{-1} W^T < 0$.

(ıı) $X < 0$, *and* $Y - W X^{-1} W^T < 0$.

2. The observer-based adaptive fault-tolerant controller design

In this section, cubic observers [21] are selected for their capability to generalize linear observers, which leads to faster response times and improved performance in nonlinear systems. Furthermore, a distributed adaptive fault-tolerant control method utilizing these observers is proposed. The formulation of the cubic observer for multi-agent systems under actuator and sensor faults is presented as follows:

$$\begin{aligned} \dot{\hat{x}}_i(t) &= A \hat{x}_i(t) + B(u_i(t) + \hat{\delta}_i^a) + f_L(\hat{x}_i(t), t) \\ &\quad + L(y_i(t) - \hat{y}_i(t)) \\ &\quad - e_{yi}^T(t) \theta e_{yi}(t) N e_{yi}(t) \\ \hat{y}_i(t) &= C \hat{x}_i(t) + \hat{\delta}_i^s \end{aligned} \quad (2)$$

where $\hat{x}_i(t)$, $\hat{y}_i(t)$, $\hat{\delta}_i^a$, $\hat{\delta}_i^s$ are the observer state vector, observer output vector, the estimations of actuator and sensor fault, respectively. $L \in \mathbb{R}^{n \times n_y}$, $N \in \mathbb{R}^{n \times n_y}$, $\theta \in \mathbb{R}^{n_y \times n_y}$ are the observer gains to be designed later. Define the state estimation error and tracking error of the *ith* follower as:

$$e_{xi}(t) = x_i(t) - \hat{x}_i(t) \quad (3)$$

$$e_{0i}(t) = x_i(t) - x_0(t) \quad (4)$$

and also actuator and sensor fault estimation errors as:

$$\begin{aligned} e_{\delta si}(t) &= \delta_i^s - \hat{\delta}_i^s \\ e_{\delta ai}(t) &= \delta_i^a - \hat{\delta}_i^a \end{aligned} \quad (5)$$

Subsequently, the design of the adaptive fault-tolerant controller using the cubic observer is outlined as follows:

$$\begin{aligned} \varepsilon_i(t) &= \sum_{j \in N_i} a_{ij} (\hat{x}_i - \hat{x}_j) + g_i(\hat{x}_i - x_0) \\ u_i(t) &= cK \varepsilon_i(t) - \hat{\delta}_i^a \end{aligned} \quad (6)$$

The adaptive update laws are described as follows:

$$\begin{aligned} \dot{\hat{\delta}}^s &= -(I_N \otimes Q_1)^{-1} (I_N \otimes \lambda_1) e_x \\ &\quad - (I_N \otimes Q_1)^{-1} \beta_1 \hat{\delta}^s \\ \dot{\hat{\delta}}^a &= (I_N \otimes Q_2)^{-1} (I_N \otimes \lambda_2) e_x \\ &\quad - (I_N \otimes Q_2)^{-1} \beta_2 \hat{\delta}^a \end{aligned} \quad (7)$$

The parameters $\beta_1, \beta_2, \lambda_1$ and λ_2 denote adaptive gains with compatible dimensions, which will be designed later. The theorem below gives the procedure for designing the proposed protocol.

Theorem 1: Consider the Lipschitz nonlinear MAS (1), under the premises assumption 1, along with the state observer (2), the control law (6), and the update laws (7). Given scalars $\gamma > 0, \varepsilon_i > 0$, if there exist positive definite and symmetric matrices P_1, P_2, Q_1, Q_2 with appropriate dimensions such that the LMI (8) is satisfied, it can be concluded that the multi-agent system achieves leader-following consensus and is uniformly ultimately bounded (UUB).

In the LMI (8), the matrices Π_{11} and Π_{22} are defined as $\Pi_{11} = I_N \otimes (A^T P_1 + P_1 A - C^T S^T - SC) + \gamma^2 \varepsilon_1^{-1}$ and $\Pi_{22} = I_N \otimes (A^T P_2 + P_2 A) + c(\mathcal{L} + G)^T \otimes R^T + c(\mathcal{L} + G) \otimes R) + \gamma^2 \varepsilon_2^{-1}$.

By considering appropriate dimensions for the parameters $Q_1, Q_2, P_1, P_2, \lambda_1, \lambda_2, \beta_1, \beta_2, S, R$, these parameters can be effectively computed using LMI (8). Consequently, by solving the LMI, the controller and observer gains can be calculated as $K = (P_2B)^{-1}R, L = {P_1}^{-1}S$.

To aid understanding and facilitate deeper analysis, a block diagram and a flowchart outlining the methodology implemented in this paper is provided (see Fig. 1 and Fig. 2).

To simplify, the time argument "t" has been omitted.

Proof. To analyze the stability of system, we consider the following Lyapunov function

$$\begin{aligned} V = e_x^T(I_N \otimes P_1)e_x + e_0^T(I_N \otimes P_2)e_0 \\ + e_{\delta s}^T(I_N \otimes Q_1)e_{\delta s} \\ + e_{\delta a}^T(I_N \otimes Q_2)e_{\delta a} \end{aligned} \tag{9}$$

The derivative of V along the system and observer is

$$\begin{aligned} \dot{V} = \dot{e}_x^T(I_N \otimes P_1)e_x + e_x^T(I_N \otimes P_1)\dot{e}_x \\ + \dot{e}_0^T(I_N \otimes P_2)e_0 \\ + e_0^T(I_N \otimes P_2)\dot{e}_0 \\ + \dot{e}_{\delta s}^T(I_N \otimes Q_1)e_{\delta s} \\ + e_{\delta s}^T(I_N \otimes Q_1)\dot{e}_{\delta s} \\ + \dot{e}_{\delta a}^T(I_N \otimes Q_2)e_{\delta a} \\ + e_{\delta a}^T(I_N \otimes Q_2)\dot{e}_{\delta a} \end{aligned} \tag{10}$$

With the definitions $f_L(x_i(t), t) - f_L(x_0(t), t) \triangleq \Delta f_{Li0}$ and $f_L(x_i(t), t) - f_L(\hat{x}_i(t), t) \triangleq \Delta f_{Li}$, Equation (10) takes the form

$$\begin{aligned} &\dot{V} \\ &= ((I_N \otimes (A - LC))e_x(t) + 1_N \otimes \Delta f_L \\ &+ (I_N \otimes B)e_{\delta a} - (I_N \otimes L)e_{\delta s} \\ &+ e_y^T(I_N \otimes \theta)e_y(I_N \otimes N)e_y)^T(I_N \otimes P_1)e_x \\ &+ e_x^T(I_N \otimes P_1)((I_N \otimes (A - LC))e_x(t) \\ &+ 1_N \otimes \Delta f_L + (I_N \otimes B)e_{\delta a} - (I_N \otimes L)e_{\delta s} \\ &+ e_y^T(I_N \otimes \theta)e_y(I_N \otimes N)e_y) \\ &+ ([(I_N \otimes A) + c(\mathcal{L} + G) \otimes BK)]e_0(t) \\ &- [c(\mathcal{L} + G) \otimes BK)]e_x + (I_N \otimes B)e_{\delta a} \\ &+ 1_N \otimes \Delta f_{L0})^T(I_N \otimes P_2)e_0 \\ &+ e_0^T(I_N \otimes P_2)([(I_N \otimes A) \\ &+ c(\mathcal{L} + G) \otimes BK)]e_0(t) \\ &- [c(\mathcal{L} + G) \otimes BK)]e_x + (I_N \otimes B)e_{\delta a} \\ &+ 1_N \otimes \Delta f_{L0}) + \dot{e}_{\delta s}^T(I_N \otimes Q_1)e_{\delta s} \\ &+ e_{\delta s}^T(I_N \otimes Q_1)\dot{e}_{\delta s} + \dot{e}_{\delta a}^T(I_N \otimes Q_2)e_{\delta a} \\ &+ e_{\delta a}^T(I_N \otimes Q_2)\dot{e}_{\delta a} \end{aligned} \tag{11}$$

From Equation (11), we obtain

$$\begin{aligned} &\dot{V} \\ &= e_x^T(I_N \otimes (A - LC)^T P_1 + I_N \otimes P_1(A - LC))e_x \\ &+ 2e_x^T(I_N \otimes P_1)\Delta f_L + 2e_x^T(I_N \otimes P_1 B)e_{\delta a} \\ &- 2{e_{\delta s}}^T(I_N \otimes L^T P_1)e_x \\ &+ e_0^T[(I_N \otimes A^T P_2 + I_N \otimes P_2 A) \\ &+ c(\mathcal{L} + G)^T \otimes K^T B^T P_2 \\ &+ c(\mathcal{L} + G) \otimes P_2 BK)]e_0 \\ &- 2e_x^T[c(\mathcal{L} + G)^T \otimes K^T B^T P_2]e_0 \\ &+ 2e_0^T(I_N \otimes P_2 B)e_{\delta a} + 2e_0^T(I_N \otimes P_2)\Delta f_{L0} \\ &+ 2e_{\delta s}^T(I_N \otimes Q_1)\dot{\delta}^s - 2e_{\delta s}^T(I_N \otimes Q_1)\,\dot{\hat{\delta}}^s \\ &+ 2e_{\delta a}^T(I_N \otimes Q_2)\dot{\delta}^a - 2e_{\delta a}^T(I_N \otimes Q_2)\,\dot{\hat{\delta}}^a \\ &+ 2e_y^T(I_N \otimes \theta)e_y e_x^T(I_N \otimes P_1 N)e_y \end{aligned} \tag{12}$$

$$\begin{bmatrix} \Pi_{11} & I_N \otimes P_1 & -c(\mathcal{L}+G)^T \otimes R^T & 0 & (I_N \otimes {\lambda_1}^T) - (I_N \otimes S) & 0 & 0 & (I_N \otimes P_1 B) - (I_N \otimes {\lambda_2}^T) & 0 & 0 \\ * & -{\varepsilon_1}^{-1} & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & \Pi_{22} & I_N \otimes P_2 & 0 & 0 & 0 & I_N \otimes P_2 B & 0 & 0 \\ * & * & * & -{\varepsilon_2}^{-1} I & 0 & 0 & 0 & 0 & 0 & 0 \\ * & * & * & * & -2\beta_1 & I_N \otimes Q_1 & \beta_1 & 0 & 0 & 0 \\ * & * & * & * & * & -{\varepsilon_3}^{-1} I & 0 & 0 & 0 & 0 \\ * & * & * & * & * & * & -{\varepsilon_6}^{-1} I & 0 & 0 & 0 \\ * & * & * & * & * & * & * & -2\beta_2 & I_N \otimes Q_2 & \beta_2 \\ * & * & * & * & * & * & * & * & -{\varepsilon_4}^{-1} I & 0 \\ * & * & * & * & * & * & * & * & * & -{\varepsilon_5}^{-1} I \end{bmatrix} \le 0 \tag{8}$$

Using Lemma 1 and the following relations,

$2e_x^T(I_N \otimes P_1)\Delta f_L \le \varepsilon_1 e_x^T(I_N \otimes P_1 P_1)e_x + \gamma^2 {\varepsilon_1}^{-1} {e_x}^T e_x$

$2e_0^T(I_N \otimes P_2)\Delta f_{L0} \le \varepsilon_2 e_0^T(I_N \otimes P_2 P_2)e_0 + \gamma^2 {\varepsilon_2}^{-1} {e_0}^T e_0$

$2e_{\delta s}^T(I_N \otimes Q_1)\dot{\delta}^s \le \varepsilon_3 e_{\delta s}^T(I_N \otimes Q_1 Q_1)e_{\delta s} + {\varepsilon_3}^{-1} {\dot{\delta}^s}{}^T \dot{\delta}^s$

$2e_{\delta a}^T(I_N \otimes Q_2)\dot{\delta}^a \le \varepsilon_4 e_{\delta a}^T(I_N \otimes Q_2 Q_2)e_{\delta a} + {\varepsilon_4}^{-1} {\dot{\delta}^a}{}^T \dot{\delta}^a$

we have

$$
\begin{aligned}
&\dot{V}\\
&\le e_x^T(I_N\otimes(A-LC)^TP_1 + I_N\otimes P_1(A-LC)\\
&+ \varepsilon_1(I_N\otimes P_1P_1) + \gamma^2{\varepsilon_1}^{-1})e_x\\
&+ 2{e_{\delta a}}^T(I_N\otimes B^TP_1)e_x - 2{e_{\delta s}}^T(I_N\otimes L^TP_1)e_x\\
&+ e_0^T[(I_N\otimes A^TP_2 + I_N\otimes P_2A)\\
&+ c(\mathcal{L}+G)^T\otimes K^TB^TP_2\\
&+ c(\mathcal{L}+G)\otimes P_2BK)+\varepsilon_2(I_N\otimes P_2P_2)\\
&+ \gamma^2{\varepsilon_2}^{-1}]e_0 - 2e_x^T[c(\mathcal{L}+G)^T\otimes K^TB^TP_2]e_0\\
&+ 2e_0^T(I_N\otimes P_2B)e_{\delta a} + \varepsilon_3 e_{\delta s}^T(I_N\otimes Q_1Q_1)e_{\delta s}\\
&+ {\varepsilon_3}^{-1}{\dot{\delta}^s}{}^T\dot{\delta}^s - 2e_{\delta s}^T(I_N\otimes Q_1)\,\dot{\hat{\delta}}^s\\
&+ \varepsilon_4 e_{\delta a}^T(I_N\otimes Q_2Q_2)e_{\delta a} + {\varepsilon_4}^{-1}{\dot{\delta}^a}{}^T\dot{\delta}^a\\
&- 2e_{\delta a}^T(I_N\otimes Q_2)\,\dot{\hat{\delta}}^a\\
&+ 2e_y^T(I_N\otimes\theta)e_y e_x^T(I_N\otimes P_1N)e_y
\end{aligned}
\tag{13}
$$

Substituting the update laws (7) shows that

$$
\begin{aligned}
&\dot{V}\\
&\le e_x^T(I_N\otimes(A-LC)^TP_1 + I_N\otimes P_1(A-LC)\\
&+ \varepsilon_1(I_N\otimes P_1P_1) + \gamma^2{\varepsilon_1}^{-1})e_x\\
&+ 2{e_{\delta s}}^T[(I_N\otimes\lambda_1) - (I_N\otimes L^TP_1)]e_x\\
&+ 2{e_{\delta a}}^T[(I_N\otimes B^TP_1) - (I_N\otimes\lambda_2)]e_x\\
&- 2e_x^T(I_N\otimes P_1L)e_{\delta s}\\
&+ e_0^T[(I_N\otimes A^TP_2 + I_N\otimes P_2A)\\
&+ c(\mathcal{L}+G)^T\otimes K^TB^TP_2\\
&+ c(\mathcal{L}+G)\otimes P_2BK)+\varepsilon_2(I_N\otimes P_2P_2)\\
&+ \gamma^2{\varepsilon_2}^{-1}]e_0 - 2e_x^T[c(\mathcal{L}+G)^T\otimes K^TB^TP_2]e_0\\
&+ 2e_0^T(I_N\otimes P_2B)e_{\delta a} + \varepsilon_3 e_{\delta s}^T(I_N\otimes Q_1Q_1)e_{\delta s}\\
&+ {\varepsilon_3}^{-1}{\dot{\delta}^s}{}^T\dot{\delta}^s - 2e_{\delta a}^T\beta_2 e_{\delta a} + 2e_{\delta a}^T\beta_2\delta^a\\
&- 2e_{\delta s}^T\,\beta_1 e_{\delta s} + 2e_{\delta s}^T\beta_1\delta^s\\
&+ \varepsilon_4 e_{\delta a}^T(I_N\otimes Q_2Q_2)e_{\delta a} + {\varepsilon_4}^{-1}{\dot{\delta}^a}{}^T\dot{\delta}^a\\
&+ 2e_y^T(I_N\otimes\theta)e_y e_x^T(I_N\otimes P_1N)e_y
\end{aligned}
\tag{14}
$$

Using Lemma 1 again for the following relations

$$2e_{\delta s}^T\beta_1\delta^s \le \varepsilon_6 e_{\delta s}^T\beta_1{\beta_1}^T e_{\delta s} + {\varepsilon_6}^{-1}{\delta^s}^T\delta^s$$
$$2e_{\delta a}^T\beta_2\delta^a \le \varepsilon_5 e_{\delta a}^T\beta_2{\beta_2}^T e_{\delta a} + {\varepsilon_5}^{-1}{\delta^a}^T\delta^a$$

We get

$$
\begin{aligned}
&\dot{V}\\
&\le e_x^T(I_N\otimes(A-LC)^TP_1 + I_N\otimes P_1(A-LC)\\
&+ \varepsilon_1(I_N\otimes P_1P_1) + \gamma^2{\varepsilon_1}^{-1})e_x\\
&+ 2{e_{\delta s}}^T[(I_N\otimes\lambda_1) - (I_N\otimes L^TP_1)]e_x\\
&+ 2{e_{\delta a}}^T[(I_N\otimes B^TP_1) - (I_N\otimes\lambda_2)]e_x\\
&- 2e_x^T(I_N\otimes P_1L)e_{\delta s}\\
&+ e_0^T[(I_N\otimes A^TP_2 + I_N\otimes P_2A)\\
&+ c(\mathcal{L}+G)^T\otimes K^TB^TP_2\\
&+ c(\mathcal{L}+G)\otimes P_2BK)+\varepsilon_2(I_N\otimes P_2P_2)\\
&+ \gamma^2{\varepsilon_2}^{-1}]e_0 - 2e_x^T[c(\mathcal{L}+G)^T\otimes K^TB^TP_2]e_0\\
&+ 2e_0^T(I_N\otimes P_2B)e_{\delta a} + \varepsilon_3 e_{\delta s}^T(I_N\otimes Q_1Q_1)e_{\delta s}\\
&+ {\varepsilon_3}^{-1}{\dot{\delta}^s}{}^T\dot{\delta}^s - 2e_{\delta a}^T\beta_2 e_{\delta a} - 2e_{\delta s}^T\,\beta_1 e_{\delta s}\\
&+ \varepsilon_4 e_{\delta a}^T(I_N\otimes Q_2Q_2)e_{\delta a} + {\varepsilon_4}^{-1}{\dot{\delta}^a}{}^T\dot{\delta}^a\\
&+ \varepsilon_5 e_{\delta a}^T\beta_2{\beta_2}^T e_{\delta a} + {\varepsilon_5}^{-1}{\delta^a}^T\delta^a\\
&+ \varepsilon_6 e_{\delta s}^T\beta_1{\beta_1}^T e_{\delta s} + {\varepsilon_6}^{-1}{\delta^s}^T\delta^s\\
&+ 2e_y^T(I_N\otimes\theta)e_y e_x^T(I_N\otimes P_1N)e_y
\end{aligned}
\tag{15}
$$

Note that

$$
\begin{aligned}
&2e_y^T(I_N\otimes\theta)e_y e_x^T(I_N\otimes P_1N)e_y\\
&= 2e_y^T[(I_N\otimes\theta)(Ce_x + e_{\delta s})e_x^T(I_N\otimes P_1N)]e_y\\
&= 2e_y^T[(I_N\otimes\theta C)e_x e_x^T(I_N\otimes P_1N)\\
&+ (I_N\otimes\theta)e_{\delta s}e_x^T(I_N\otimes P_1N)]e_y
\end{aligned}
\tag{16}
$$

by using Lemma 1

$$
\begin{aligned}
&(I_N\otimes\theta)e_{\delta s}e_x^T(I_N\otimes P_1N)\\
&\le \varepsilon_7(I_N\otimes\theta)e_{\delta s}{e_{\delta s}}^T(I_N\otimes\theta^T)\\
&+ {\varepsilon_7}^{-1}(I_N\otimes N^TP_1)e_x e_x^T(I_N\otimes P_1N)
\end{aligned}
$$

equation (16) leads to the following form

$$
\begin{aligned}
&(I_N\otimes\theta C)e_x e_x^T(I_N\otimes P_1N)\\
&+ {\varepsilon_7}^{-1}(I_N\otimes N^TP_1)e_x e_x^T(I_N\otimes P_1N)\\
&+ \varepsilon_7(I_N\otimes\theta)e_{\delta s}{e_{\delta s}}^T(I_N\otimes\theta^T)\\
&= [((I_N\otimes\theta C)\\
&+ {\varepsilon_7}^{-1}(I_N\otimes N^TP_1))]e_x e_x^T(I_N\otimes P_1N)\\
&+ \varepsilon_7(I_N\otimes\theta)e_{\delta s}{e_{\delta s}}^T(I_N\otimes\theta^T)\\
&= 2e_y^T((I_N\otimes\theta C)\\
&+ {\varepsilon_7}^{-1}(I_N\otimes N^TP_1))e_x e_x^T(I_N\otimes P_1N)e_y\\
&+ 2\varepsilon_7 e_y^T(I_N\otimes\theta)e_{\delta s}{e_{\delta s}}^T(I_N\otimes\theta^T)e_y
\end{aligned}
\tag{17}
$$

Applying (17) to (15) and simplifying gives

$$
\begin{aligned}
&\dot{V}\\
&\le e_x^T(I_N\otimes(A-LC)^TP_1 + I_N\otimes P_1(A-LC)\\
&+ \varepsilon_1(I_N\otimes P_1P_1) + \gamma^2{\varepsilon_1}^{-1})e_x\\
&+ 2{e_{\delta s}}^T[(I_N\otimes\lambda_1) - (I_N\otimes L^TP_1)]e_x\\
&+ 2{e_{\delta a}}^T[(I_N\otimes B^TP_1) - (I_N\otimes\lambda_2)]e_x\\
&+ e_0^T[(I_N\otimes A^TP_2 + I_N\otimes P_2A)\\
&+ c(\mathcal{L}+G)^T\otimes K^TB^TP_2\\
&+ c(\mathcal{L}+G)\otimes P_2BK)+\varepsilon_2(I_N\otimes P_2P_2)\\
&+ \gamma^2{\varepsilon_2}^{-1}]e_0 - e_x^T[c(\mathcal{L}+G)^T\otimes K^TB^TP_2]e_0\\
&+ e_{\delta s}^T\big(\varepsilon_3(I_N\otimes Q_1Q_1) - 2\,\beta_1 + \varepsilon_6\beta_1{\beta_1}^T\big)e_{\delta s}\\
&+ e_{\delta a}^T\big(\varepsilon_4(I_N\otimes Q_2Q_2) + \varepsilon_5\beta_2{\beta_2}^T - 2\beta_2\big)e_{\delta a}\\
&+ 2e_0^T(I_N\otimes P_2B)e_{\delta a} + ({\varepsilon_5}^{-1}){\delta^a}^T\delta^a\\
&+ {\varepsilon_6}^{-1}{\delta^s}^T\delta^s + {\varepsilon_4}^{-1}{\dot{\delta}^a}{}^T\dot{\delta}^a + {\varepsilon_3}^{-1}{\dot{\delta}^s}{}^T\dot{\delta}^s\\
&+ 2{\varepsilon_7}^{-1}e_y^T(I_N\otimes\theta)e_{\delta s}{e_{\delta s}}^T(I_N\otimes\theta^T)e_y\\
&+ 2e_y^T[(I_N\otimes\theta C)\\
&+ \varepsilon_7(I_N\otimes N^TP_1)]e_x e_x^T(I_N\otimes P_1N)e_y
\end{aligned}
\tag{18}
$$

By considering $N = -\varepsilon_7{P_1}^{-1}C^T\theta^T$, (18) can be transformed as

$$
\begin{aligned}
&\dot{V} \\
&\leq e_x^T(I_N \otimes (A-LC)^T P_1 + I_N \otimes P_1(A-LC) \\
&+ \varepsilon_1 (I_N \otimes P_1 P_1) + \gamma^2 {\varepsilon_1}^{-1}) e_x \\
&+ 2 {e_{\delta s}}^T [(I_N \otimes \lambda_1) - (I_N \otimes L^T P_1)] e_x \\
&+ 2 {e_{\delta a}}^T [(I_N \otimes B^T P_1) - (I_N \otimes \lambda_2)] e_x \\
&+ e_0^T [I_N \otimes (A^T P_2 + P_2 A) \\
&+ c(\mathcal{L}+G)^T \otimes K^T B^T P_2 \\
&+ c(\mathcal{L}+G) \otimes P_2 BK) + \varepsilon_2 (I_N \otimes P_2 P_2) \\
&+ \gamma^2 {\varepsilon_2}^{-1}] e_0 - 2 e_x^T [c(\mathcal{L}+G)^T \otimes K^T B^T P_2] e_0 \\
&+ e_{\delta s}^T (\varepsilon_3 (I_N \otimes Q_1 Q_1) - 2\beta_1 + \varepsilon_6 \beta_1 {\beta_1}^T) e_{\delta s} \\
&+ e_{\delta a}^T (\varepsilon_4 (I_N \otimes Q_2 Q_2) + \varepsilon_5 \beta_2 {\beta_2}^T - 2\beta_2) e_{\delta a} \\
&+ 2 e_0^T (I_N \otimes P_2 B) e_{\delta a} + {\varepsilon_3}^{-1} {\dot{\delta}^s}{}^T \dot{\delta}^s \\
&+ {\varepsilon_4}^{-1} {\dot{\delta}^a}{}^T \dot{\delta}^a + {\varepsilon_5}^{-1} {\delta^a}^T \delta^a + {\varepsilon_6}^{-1} {\delta^s}^T \delta^s \\
&+ 2 {\varepsilon_7}^{-1} e_y^T (I_N \otimes \theta) e_{\delta s} {e_{\delta s}}^T (I_N \otimes \theta^T) e_y
\end{aligned} \tag{19}
$$

By incorporating assumption 1, equation (19) can be obtained as

$$
\dot{V} \leq \begin{bmatrix} e_x^T \\ e_0^T \\ e_{\delta s}^T \\ e_{\delta a}^T \end{bmatrix}^T \begin{bmatrix} \psi_{11} & \psi_{12} & \psi_{13} & \psi_{14} \\ \psi_{21} & \psi_{22} & 0 & \psi_{24} \\ \psi_{31} & 0 & \psi_{33} & 0 \\ \psi_{41} & \psi_{42} & 0 & \psi_{44} \end{bmatrix} \begin{bmatrix} e_x \\ e_0 \\ e_{\delta s} \\ e_{\delta a} \end{bmatrix} + \sigma \tag{20}
$$

Equation (20) can be expressed as

$$
\dot{V} \leq \bar{X}^T \Psi \bar{X} + \sigma \tag{21}
$$

Where

$$
\bar{X} = [e_x \quad e_0 \quad e_{\delta s} \quad e_{\delta a}]^T ,
$$

$$
\begin{aligned}
\sigma \triangleq\ & 2{\varepsilon_7}^{-1} e_y^T (I_N \otimes \theta) e_{\delta s} {e_{\delta s}}^T (I_N \otimes \theta^T) e_y + {\varepsilon_3}^{-1} {S_2}^2 \\
& + {\varepsilon_4}^{-1} {A_2}^2 + {\varepsilon_5}^{-1} {A_1}^2 + {\varepsilon_6}^{-1} {S_1}^2 \\
& > 0
\end{aligned}
$$

and Ψ is given as

$$
\begin{aligned}
\psi_{11} &= I_N \otimes (A-LC)^T P_1 + I_N \otimes P_1 (A-LC) \\
&\quad + \varepsilon_1 (I_N \otimes P_1 P_1) + \gamma^2 {\varepsilon_1}^{-1} \\
\psi_{12} &= -c(\mathcal{L}+G)^T \otimes K^T B^T P_2 \\
\psi_{13} &= (I_N \otimes {\lambda_1}^T) - (I_N \otimes P_1 L) \\
\psi_{14} &= (I_N \otimes P_1 B) - (I_N \otimes {\lambda_2}^T) \\
\psi_{21} &= -c(\mathcal{L}+G) \otimes P_2 BK \\
\psi_{22} &= I_N \otimes (A^T P_2 + P_2 A) + c(\mathcal{L}+G)^T \otimes K^T B^T P_2 \\
&\quad + c(\mathcal{L}+G) \otimes P_2 BK) + \varepsilon_2 (I_N \otimes P_2 P_2) \\
&\quad + \gamma^2 {\varepsilon_2}^{-1} \\
\psi_{24} &= (I_N \otimes P_2 B) \\
\psi_{31} &= (I_N \otimes \lambda_1) - (I_N \otimes L^T P_1) \\
\psi_{33} &= \varepsilon_3 (I_N \otimes Q_1 Q_1) - 2\beta_1 + \varepsilon_6 \beta_1 {\beta_1}^T \\
\psi_{41} &= (I_N \otimes B^T P_1) - (I_N \otimes \lambda_2) \\
\psi_{42} &= (I_N \otimes B^T P_2) \\
\psi_{44} &= \varepsilon_4 (I_N \otimes Q_2 Q_2) - 2\beta_2 + \varepsilon_5 \beta_2 {\beta_2}^T
\end{aligned}
$$

It follows that

$\dot{V} \leq \alpha V + \sigma$

By considering ${\varepsilon_i}^{-1}$ for $i = 3, \ldots, 7$ to be small, the value of σ is correspondingly small. This implies that the rate of change of V, denoted as $\dot{V}$, remains uniformly ultimately bounded. Consequently, it can be stated that

$\dot{V} \leq -\lambda_{min}(\alpha) \| \bar{X} \|^2 + \sigma$

where $\lambda_{min}(\alpha)$ is the minimum eigenvalue of matrix Ψ. If $\lambda_{min}(\alpha) \| \bar{X} \|^2 > \sigma$, then $\dot{V} < 0$, and according to Theorem 1, the state estimation error, tracking error, and fault estimation errors remain bounded. Utilizing the Schur Complement Lemma, the equivalent constraint matrix ψ in (21) can be expressed as LMI (8). Consequently, it follows from Lyapunov stability theory that the multi-agent system (1) is uniformly ultimately bounded and achieves leader-following consensus. For clarity, the notations used in the mathematical formulations are listed in Table 2.

The proof is completed.

Table 2. Notations used in the mathematical formulations

Notations	**Description**
δ^a, δ^s	Actuator/ Sensor fault
$\dot{\delta}^a, \dot{\delta}^s$	Derivative of actuator/sensor fault
$\hat{\delta}^a, \hat{\delta}^s$	Actuator/Sensor fault estimation
e_{xi}, e_{0i}	State estimation/ Tracking error
$e_{\delta ai}, e_{\delta si}$	Actuator/Sesnsor fault estimation error
c	Scalar constant
K	Controller design parameters
L, N, θ	Observer design parameters
$\lambda_1, \lambda_2, \beta_1, \beta_2$	Fault estimation adaptive gains
P_1, P_2, Q_1, Q_2	Positive definite and symmetric Lyapunov matrices

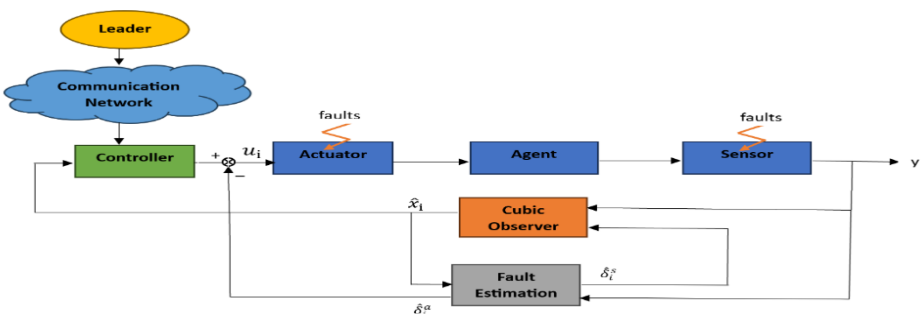

Fig 1. Block diagram of agent i with the observer-based adaptive FTC control

3. Simulation

This section presents two examples to illustrate the effectiveness of the proposed design approach. While the cubic-based estimation framework has been successfully validated for linear MAS in our previous study [26]—where comparative analyses between cubic and linear observers regarding the mean tracking error were conducted—the current simulation study focuses on nonlinear Lipschitz-type MAS. To ensure a rigorous and meaningful evaluation, the proposed method is benchmarked against Reference [24], which provides a comparable framework for nonlinear Lipschitz systems under similar fault conditions. The LMI (8) is solved using the YALMIP toolbox in MATLAB R2021b to compute the design parameters, and the simulation results are presented as follows:

Example 1. This example, adapted from reference [24], considers a network of five satellite vehicles, consisting of one leader (designated as index 0) and four followers (indexed as $i = 1,2,3,4$). The results obtained from the proposed method are compared against those presented in [24]. The system matrices are described as follows:

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 2.94 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad f_L(x) = \begin{bmatrix} 0.1 \sin x_1 \\ 0 \end{bmatrix}$$

Here, the states x_1 and x_2 represent the pitch angle and pitch rate, respectively. The directed communication topology is depicted in Fig. 3.

Scenario 1: The control protocols are implemented based on Theorem 1 and by solving the linear matrix inequality (10) and selecting $c = 1$ *and* $\gamma = 0.1$, the design parameters are determined as $K = [-18.36 \quad -33.76]$, $L = [1.27 \quad 3.07]^T$, $\lambda_2 = [-1.02 \quad 6.03]$, $Q_2 = 0.0956$, $\beta_2 = 0.1008$. In this scenario, we assume that $\delta_i^s = 0$, indicating that no sensor faults occur. Two distinct types of time-varying actuator faults, along with the disturbance considered in [24], are taken into account. The simultaneous faults in the first and third actuators are as follows:

$$\delta_1^a(t) = \begin{cases} 0 & 0 \le t < 2 \\ 0.5 - 0.25t & 2 \le t < 6 \\ -1 & t \ge 6 \end{cases},$$

$$\delta_3^a(t) = \begin{cases} 0 & 0 \le t < 4 \\ 1 + (t-4)e^{-0.5(t-4)} & t \ge 4 \end{cases}$$

$$\delta_2^a = \delta_4^a = 0$$

The initial states of the leaders and followers are selected to match those in [24]. The simulation results for fault estimation are illustrated in Figs. 4 and 5, demonstrating the system's effectiveness in identifying and estimating time-varying faults within the proposed framework.

Response curves of $x_{i1}(t)$ and $x_{i2}(t)$ using proposed adaptive fault-tolerant control scheme in this paper are shown in Figs. 6 and 7. These curves demonstrate the effectiveness of the proposed controller in reducing the impact of actuator faults on the state trajectories of agents, while ensuring consensus is achieved. To provide a clearer and more precise comparison, Tables 3 and 4 present the absolute tracking errors observed for agents 1 and 3, using the active FTTC method in [24] and the approach proposed in this paper. As shown in Tables 3 and 4, our method demonstrates superior performance compared to active fault-tolerant control under the same conditions, particularly in the presence of severe faults, indicating better fault tolerance overall. In Table 5, we compare the performance of our method with the approach presented in [24], utilizing the Root Mean Square Error (RMSE) index. This comparison is conducted for a network of five satellite vehicles in this scenario. The results clearly demonstrate the effectiveness of our method in reducing RMSE, showcasing significantly improved accuracy in satellite vehicle performance evaluation under fault conditions.

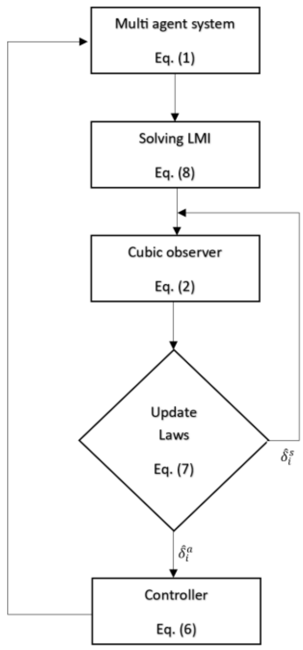

Fig 2. Flowchart of the proposed observer-based adaptive FTC control

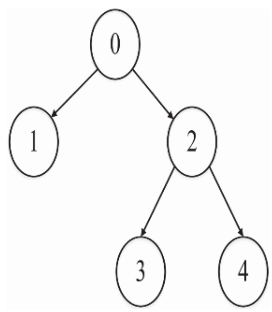

Fig 3. Communication topology of agents[24]

Scenario 2: In this case, like the first scenario, simultaneous actuator faults are considered in the first and third actuators. Additionally, a sensor fault $\delta_1^s = \sin(t)$ for $t \ge 0$ is assumed to be occur. The design parameters are determined as $c = 1, K = [-18.36 \quad -33.76], L = [1.27 \quad 3.07]^T, \lambda_1 = [8.52 \quad 5.82], \lambda_2 =$

$[-1.02 \quad 6.03]$, $Q_1 = 0.1058$, $Q_2 = 0.0956$, $\beta_1 = 0.1010$, $\beta_2 = 0.1008$. The estimation of the sensor fault occurring in agent 1 is presented in Fig. 8, showcasing the system's ability to accurately detect and estimate the faults under the proposed control strategy. Similarly, the actuator fault estimations are performed as in the previous scenario. Figs. 9 and 10 illustrate the trajectories of agents under the effect of simultaneous sensor and actuator faults. As can be seen from in these figures, leader-following consensus is achieved for nonlinear MASs despite the presence of faults. At the time of the occurrence of the sensor fault, in addition to the actuator faults, due to the fast and accurate fault diagnosis, the MAS can maintain its consensus under the control of the proposed FTC without being affected by faults. The control inputs are also illustrated in Figures 11 and 12. This illustration helps to clarify how these inputs interact with the overall system, allowing for a better understanding of their role in the control process.

Example 2. A numerical example is provided here to illustrate the effectiveness of the theoretical results in the presence of simultaneous faults. System parameters of multi-agent system are given as

$$A = \begin{bmatrix} -2 & -0.03 & 1.9 \\ 5.3 & -2 & -5 \\ -5 & 1.9 & -3.5 \end{bmatrix}, \quad B = \begin{bmatrix} -4 \\ 5 \\ -0.16 \end{bmatrix}$$

$$C = [1 \quad 1 \quad 1], \quad f_L(x) = \begin{bmatrix} 0 \\ 0 \\ 2sinx_1 \end{bmatrix}$$

The communication topology is given in Fig. 3. By selecting $c = 1$, and $\gamma = 2$, the design parameters obtained from solving the LMI in Theorem 1 are as

$K = [42.50 \quad 6.23 \quad -15.21]$,

$L = [22.65 \quad 48.12 \quad -11.14]^T$,

$\lambda_1 = [-2.50 \quad 6.25 \quad -4.39]$,

$\lambda_2 = [-12.70 \quad 7.06 \quad 0.91]$,

$Q_1 = 0.0224$, $Q_2 = 0.0033$,

$\beta_1 = 0.0114$, $\beta_2 = 0.0114$.

We demonstrate the results under multiple sensor and actuator faults, showing the reliability of our fault estimation scheme. Actuator faults considered as $\delta_1^a(t) = \sin(t)$ for $15 \le t \le 20$, $\delta_2^a(t) = 0$, $\delta_3^a(t) = sin(0.5 * t)$ for $15 \le t \le 20$ and $\delta_4^a(t) = e^{-0.1*(t-2)}$ for $t \ge 0$. The sensor faults occurring in the system is in the form of $\delta_1^s(t) = 0$, $\delta_2^s(t) = 1$ for $t \ge 0$, $\delta_3^s(t) = (t - 10)e^{-0.5*(t-10)}$ for $10 \le t \le 15$ and $\delta_4^s(t) = 0.4 * sin(0.3 * t) * cos(0.4 * t)$ for $t \ge 0$.

The simulation results for actuator and sensor fault estimation are presented in Figs. 13 and 14. It can be observed that the designed fault estimation scheme achieves accurate online estimation and the sensor and actuator faults of agents are effectively detected. The trajectories of x_{ij} which shows that all the followers track the leader via the prposed adaptive consensus protocol are shown in Fig.15. As the consensus is achieved, it is evident that the followers' states converge to the leader's state.

The simulation results demonstrate the effectiveness of the proposed cubic observer in the presence of simultaneous faults. While the cubic structure introduces a marginal increase in computational complexity compared to conventional linear observers due to the higher-order nonlinear terms, this trade-off is well-justified by the significant enhancement in estimation accuracy and the simultaneous convergence of both state and fault-related errors. Furthermore, given the theoretically proven Uniformly Ultimately Bounded (UUB) stability of the system, the observed performance in the simulations is consistent with the predicted error bounds, confirming that the observer remains robust within the specified operational parameters.

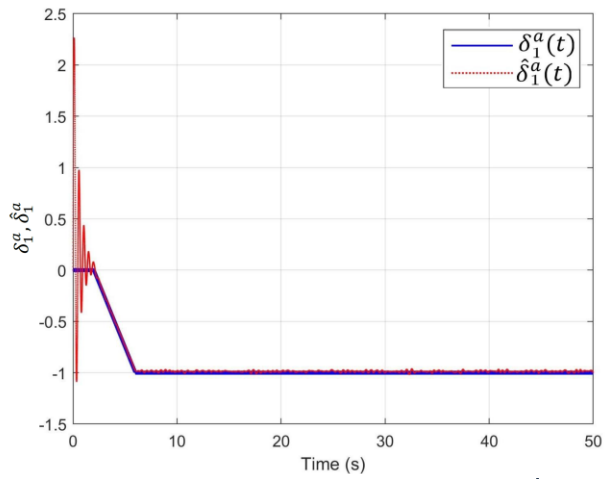

Fig 4. Actuator fault $\delta_1^a(t)$ and its estimation $\hat{\delta}_1^a(t)$

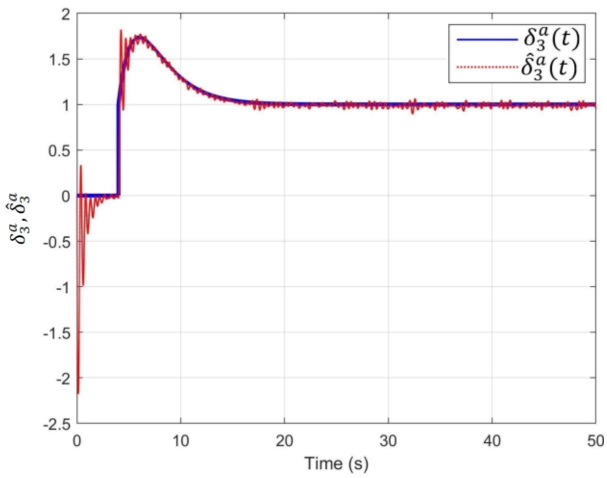

Fig 5. Actuator fault $\delta_3^a(t)$ and its estimation $\hat{\delta}_3^a(t)$

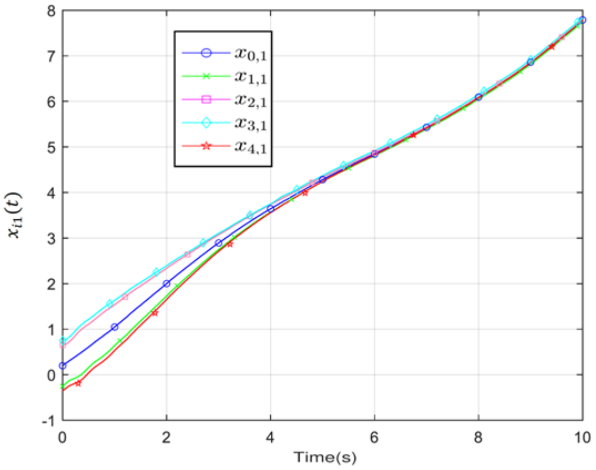

Fig 6. State trajectories of $x_{i1}(t)$ (Scenario 1)

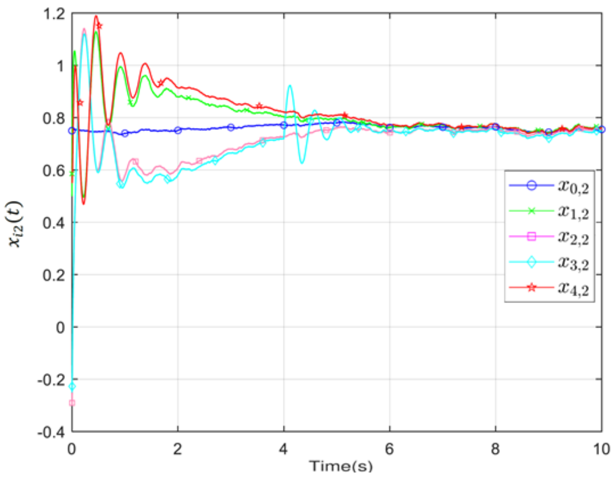

Fig 7. State trajectories of $x_{i2}(t)$ (Scenario 1)

Table 3. Absolute Tracking Errors for agent 1: Proposed Method vs. Active FTTC[24]

Time(s)	*4*	*4.5*	*5*	*5.5*	*6*	*6.5*	*7*	*7.5*	*8*	*8.5*	*9*	*9.5*
Our proposed method	*0.0799*	*0.0618*	*0.0517*	*0.0417*	*0.0341*	*0.0357*	*0.0344*	*0.03480*	*0.03743*	*0.03255*	*0.03274*	*0.03979*
Active FTTC[24]	*0.1161*	*0.1119*	*0.1256*	*0.1714*	*0.2069*	*0.1955*	*0.1353*	*0.08647*	*0.05531*	*0.04671*	*0.03716*	*0.03700*

Table 4. Absolute Tracking Errors for agent 3: Proposed Method vs. Active FTTC[24]

Time(s)	*4*	*4.5*	*5*	*5.5*	*6*	*6.5*	*7*	*7.5*	*8*	*8.5*	*9*	*9.5*
Our proposed method	*0.11422*	*0.1026*	*0.0950*	*0.0837*	*0.0673*	*0.0610*	*0.0574*	*0.0538*	*0.0547*	*0.046*	*0.03746*	*0.04056*
Active FTTC[24]	*0.06811*	*0.4410*	*0.7796*	*0.6519*	*0.3371*	*0.2216*	*0.2055*	*0.1935*	*0.1548*	*0.7067*	*0.02471*	*0.05318*

Table 5. Root Mean Square Error (RMSE): Proposed Method vs. Active FTTC[24]

	RMSE for agent 1	*RMSE for agent 3*
Our proposed method	*0.0414*	*0.0669*
Active FTTC[24]	*0.0974*	*0.3145*

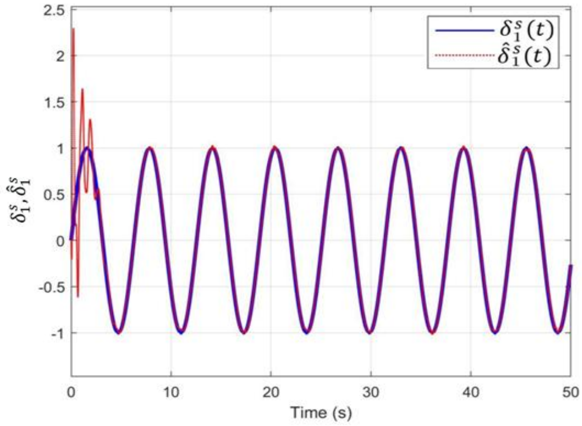

Fig 8. Sensor fault $\delta_1^s(t)$ and its estimation $\hat{\delta}_1^s(t)$

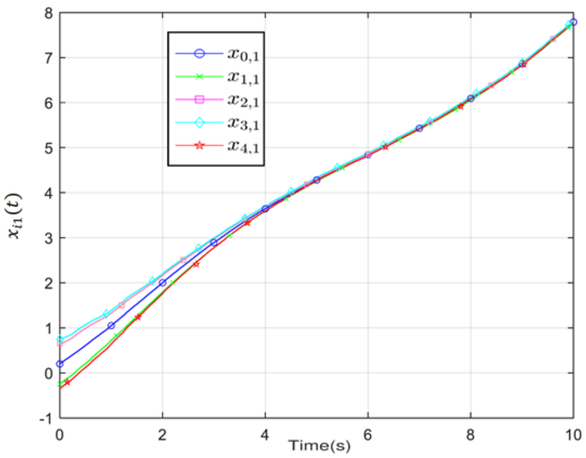

Fig 9. State trajectories of $x_{i1}(t)$ under sensor and actuator faults (Scenario 2)

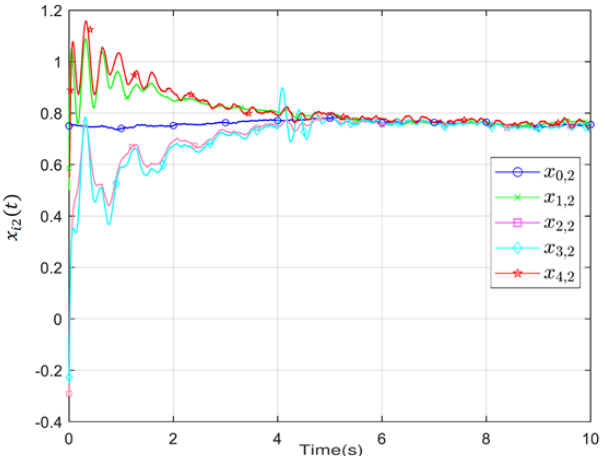

Fig 10. State trajectories of $x_{i2}(t)$ under sensor and actuator faults (Scenario 2)

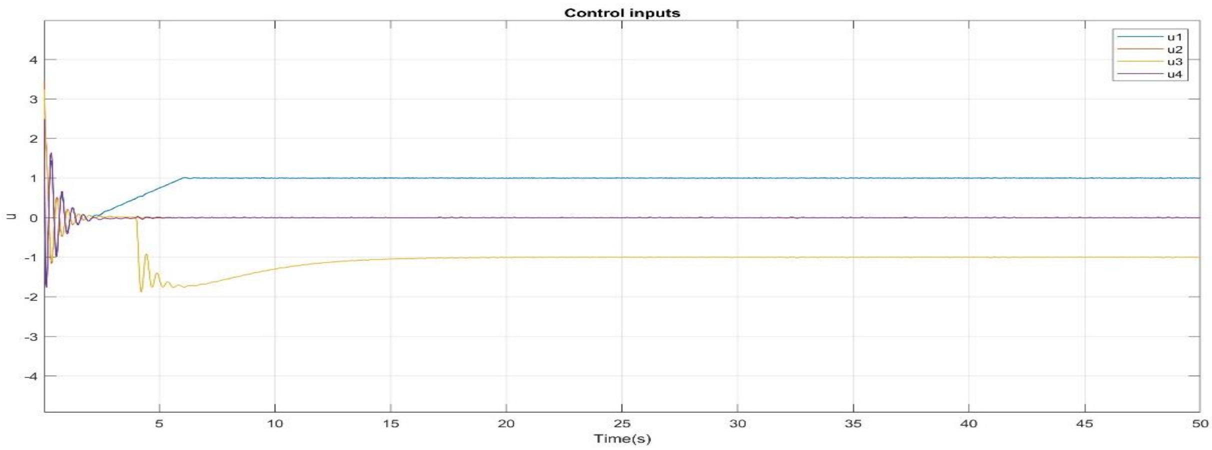

Fig 11. Control inputs of agents (Scenario 1)

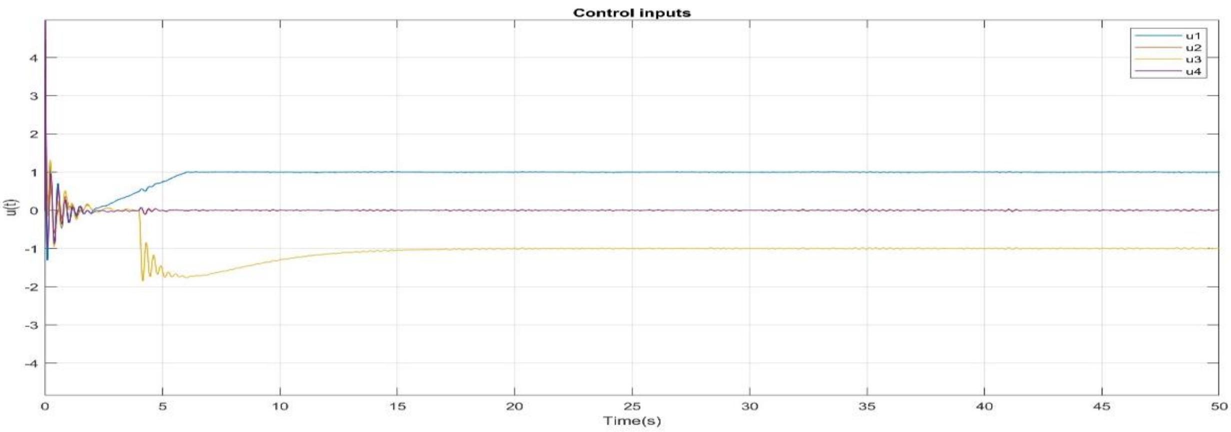

Fig 12. Control inputs of agents (Scenario 2)

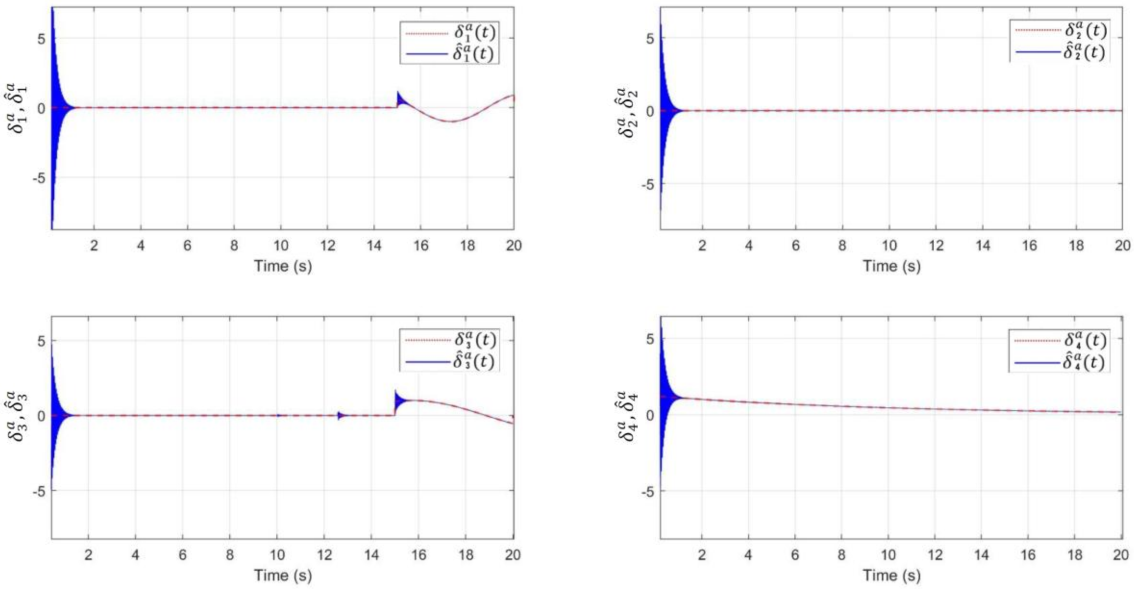

Fig 13. Actuator faults and their estimations

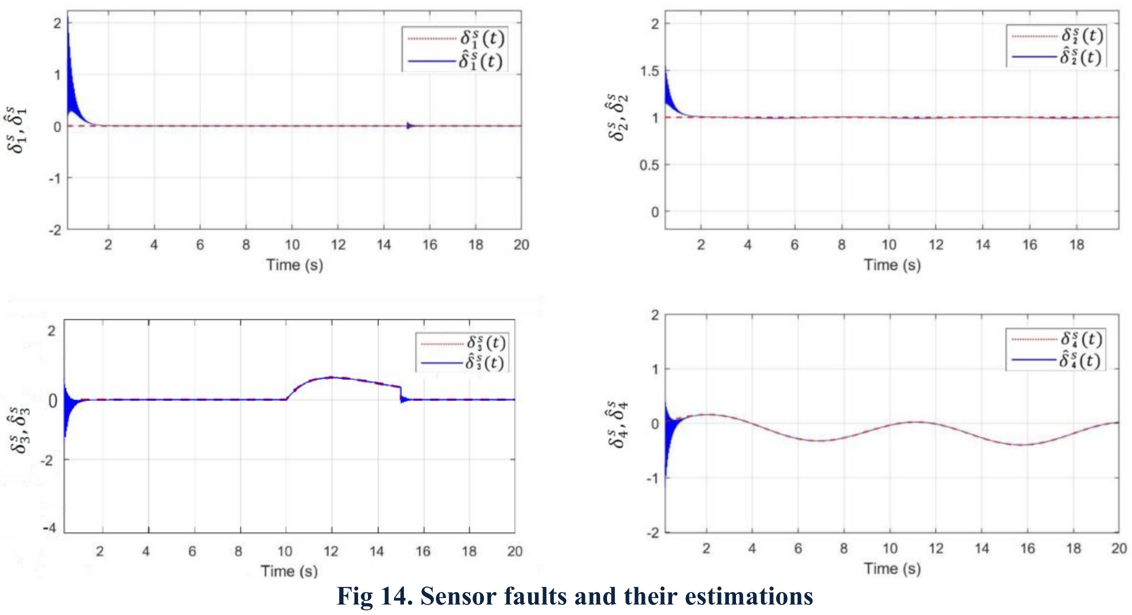

Fig 14. Sensor faults and their estimations

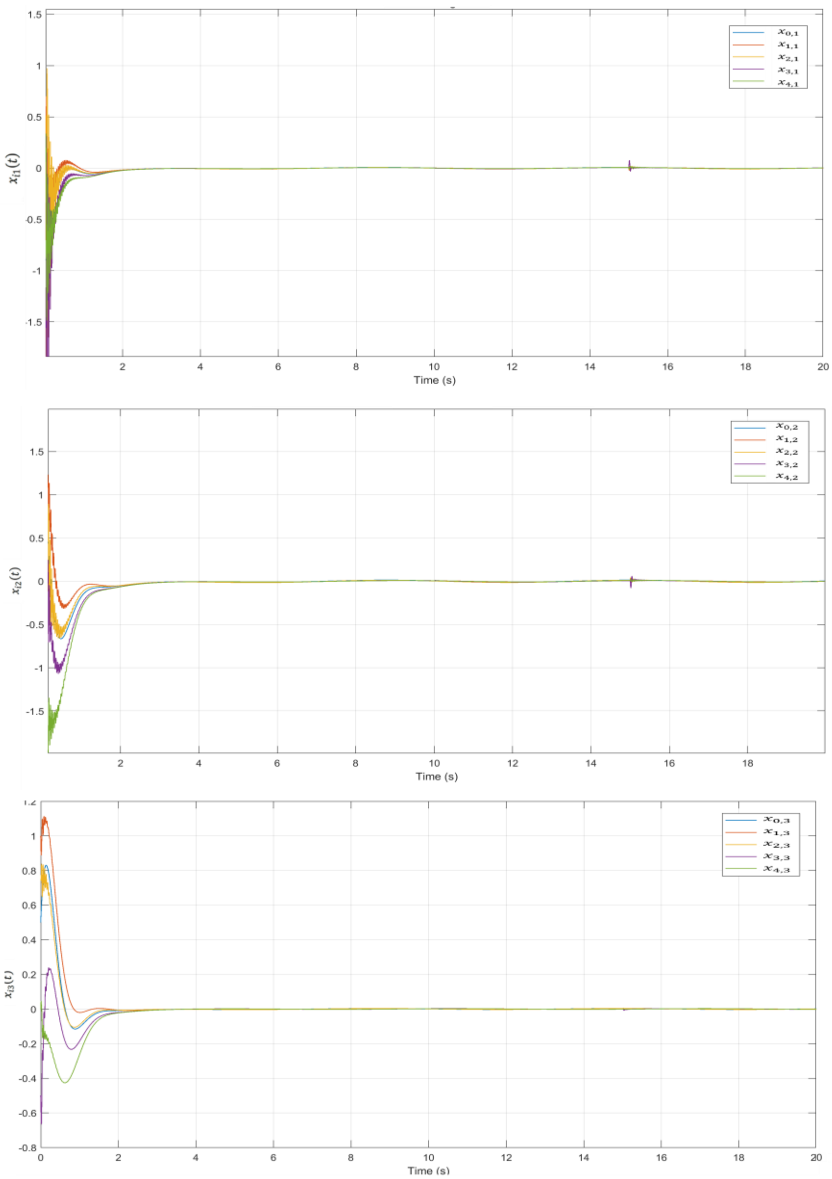

Fig 15. State trajectories of $x_{ij}(t)$ under sensor and actuator faults

4. Conclusion

This study presents an adaptive fault-tolerant consensus control strategy for Lipschitz nonlinear leader-following multi-agent systems. By utilizing cubic observers, we achieved precise state estimation, which is critical for enhancing fault detection and improving overall system performance under adverse conditions. Our approach effectively addresses simultaneous time varying sensor and actuator faults, maintaining system stability and ensuring that all agents reach consensus. The solution of Linear Matrix Inequalities (LMIs) provided the necessary parameters for effective control. Notably, numerical simulations demonstrate a significant reduction in tracking errors using our method, achieving a lower Root Mean Square Error (RMSE) compared to existing approach. These results confirm the resilience of our consensus tracking control strategy, maintaining performance even under challenging conditions. Overall, our findings highlight the effectiveness of integrating cubic observers with adaptive control in enhancing fault tolerance and system reliability.

Declarations:

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Authors' Contributions

Mahsa Hasanshahi: Conceptualization, Methodology, System Modeling, Simulation, Writing – Original Draft Preparation.

Malihe Maghfoori Farsangi: Supervision, Validation, Project Administration, Writing – Review & Editing.

Elham Amini Boroujeni: Formal Analysis, Technical Guidance, and Writing – Review & Editing.

All authors have read and approved the final manuscript.

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