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# Multi-Event-Triggered Control for Mixed Zero-Sum Games of Linear Systems

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ABSTRACT

This paper develops a multi-event-triggered (MET) control for multiplayer mixed zero-sum (MZS) differential games of linear systems. The game consists of two groups of controllers: a group of non-competitive players with distinct performance objectives, and a competitive group whose players jointly compete with a maximizing adversarial disturbance over a shared value function. Together, these two groups form the nonzero-sum (NZS) component of the game, while the competitive group and the disturbance constitute the zero-sum (ZS) interaction. The proposed approach guarantees exponential stability while enabling players to employ distinct triggering rules, leading to a decentralized MET control architecture with asynchronous control updates and reduced communication and computational burden. Leveraging Lyapunov stability analysis, a global triggering condition is first derived, where the functionals of group measurement errors are constrained by a global triggering budget. Subsequently, sufficient conditions to derive local triggering rules are established. A hierarchical budget allocation is further proposed to systematically distribute the global triggering threshold between groups and then among the players. Zeno behavior exclusion is established, and simulation results on a two-area interconnected power system load frequency control demonstrate the effectiveness of the proposed approach. Comparative studies with two centralized event-triggered methods show that the proposed approach achieves similar regulation performance while requiring fewer policy updates and enabling asynchronous triggering.

## 1. Introduction

Differential games have been widely used for modeling and solving interactive decision-making problems and have attracted increasing attention within the control theory community in recent years. Conventionally, differential games are broadly categorized into zero-sum (ZS) and nonzero-sum (NZS) formulations [1]. In ZS games, the optimal solution is characterized by a Nash saddle point, where one player seeks to minimize a given performance index while the adversarial player attempts to maximize it. Finding the Nash saddle point requires solving Hamilton-Jacobi-Isaacs (HJI) equation. Since analytic solutions are generally difficult to obtain, reinforcement learning (RL) algorithms are formulated within a control-theoretic framework to find near optimal solutions [2], [3]. While most existing studies on ZS games focus on two-player formulations, recent developments have extended the framework to multiplayer settings, where several players jointly minimize a shared objective against one or multiple adversarial players [4]. Unlike ZS games, NZS games involve players with distinct objectives that are not necessarily completely conflicting. The optimal solution of such problems is expressed in terms of a Nash equilibrium, corresponding to solving a set of coupled Hamilton Jacobi (HJ) equations or the generalized algebraic Riaccati equations (GAREs) in the case of linear dynamics. Since analytic solutions are rarely available, extensive research efforts over the past decade have focused on developing iterative algorithms [5], [6].

A recently emerging class of differential games, referred to as mixed zero-sum (MZS) games, has attracted increasing research attention due to its ability to simultaneously capture characteristics of both ZS and

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NZS formulations within a unified framework. In most existing MZS formulations, a group of players forms an NZS game, while one of the players together with an adversarial disturbance constitutes the ZS component of the problem. Consequently, MZS games inherit properties from both ZS and NZS games while introducing additional analytical and computational challenges. This class of games was first introduced in [7], where a policy iteration (PI) algorithm was developed to obtain approximate Nash solutions. Subsequent developments on MZS games can be found in [8]-[11], where different solution approaches are developed for systems involving interacting players and adversarial disturbances.

While having significant impact, the aforementioned approaches mainly rely on time-triggered control strategies. With the increasing deployment of networked control systems, event-triggered (ET) control methods have attracted considerable attention. Compared with conventional time-triggered implementations, ET control schemes can substantially reduce communication load and computational effort [12]. ET control methods have been investigated for both ZS and NZS differential games in [13]- [16]. More recently, ET strategies for MZS games were studied in [17]-[19]. In [17], a static event-triggered control (SETC) scheme is proposed while accounting for time delays affecting a subset of the players. In [18], a dynamic event-triggered control (DETC) method is developed for a MZS game involving two controllers and one adversarial player. The approach introduces an internal dynamic variable into the triggering condition to enlarge the triggering threshold. The DETC method is further developed in [19] to accommodate multiple non-competitive players while also multiple internal dynamic variables are incorporated into the triggering mechanism to further reduce policy updates.

The above-mentioned ET control approaches for differential games commonly rely on a single triggering rule shared among all players, resulting in synchronous control updates. Such centralized triggering architectures may lead to excessive signal transmissions and unnecessary controller updates, while also increasing sensitivity to network-induced imperfections including packet losses. To address these limitations, a decentralized multi-event-triggered (MET) control framework for NZS games of linear systems was proposed in [20], where each player employs an individual triggering condition, enabling asynchronous control updates and more efficient bandwidth utilization. However, the method is restricted to NZS games and does not account for the mixed game structure arising in MZS differential games

Motivated by these developments, this paper extends MET control to MZS differential games, where a group of competitive players jointly forms a ZS game against a maximizing adversarial disturbance, while simultaneously participating in an NZS interaction with non-competitive players. Consequently, only the competitive players directly compete with the adversarial player, whereas the non-competitive players simultaneously preserve their individual performance objectives. Consequently, the stability analysis simultaneously accounts for heterogeneous objectives and the coexistence of event-triggered controllers with a continuous-time maximizing adversarial player. This mixed structure, together with the coupled HJ and HJI interactions and the temporal heterogeneity between event-triggered controllers and the continuous-time maximizing adversarial player, introduces challenges in the triggering and stability analyses that are absent in standard NZS formulations.

To address these challenges, the proposed MET control approach establishes stability guarantees under a global triggering condition that incorporates the mixed HJ/HJI interactions and allows individual event-triggered players to adopt distinct local measurement errors while also accounting for the continuous-time maximizing adversarial contribution. Then, sufficient local triggering conditions for individual controllers are established by distributing the global triggering budget through local allocation weights satisfying a distribution constraint. Furthermore, a hierarchical allocation mechanism is proposed, where the global budget is distributed into group and then local budgets, providing a systematic procedure for constructing feasible local budget-allocation weights while offering additional flexibility to incorporate system information and network constraints into the triggering design. Representative feasible allocation examples are provided, where the group budgets are determined by the state weighting and disturbance-related matrices, while the local budgets are determined by the triggering costs of non-competitive players and the directional alignment of competitive players with the adversarial input. Finally, Zeno behavior exclusion is established by deriving strictly positive lower bounds on the local inter-event times (IETs) of all the controllers involved in the game.

The paper is organized as follows. Section 2 presents the formulation of the MZS game and the corresponding time-triggered Nash equilibrium solution. Section 3 introduces the proposed MET control architecture, global triggering rule, and stability analysis. Local triggering rule design and Zeno analysis are provided in Sections 4 and 5, respectively. Simulation results for a two-area interconnected LFC system and the comparative studies are presented in Section 6, followed by concluding remarks in Section 7.

## 2. Preliminaries

Consider the following MZS differential game of linear systems consisting of two groups of non-competitive and competitive control inputs:

$$\dot{x} = Ax + \sum_{j \in \mathcal{M}} B_j \, u_j + \sum_{l \in \mathcal{L}} C_l \, w_l + E\, d, \qquad (1)$$

where $x \in \mathbb{R}^n$ is the system state. The non-competitive and competitive players are represented by $u_j \in \mathbb{R}^{m_j}$ , . $j \in \mathcal{M} = \{1, \dots, M\}$ and $w_l \in \mathbb{R}^{m_l}$ , $l \in \mathcal{L} = \{1, \dots, L\}$ , respectively. The disturbance is represented by $d \in \mathbb{R}^{m_d}$ viewed as the adversarial player. In line with [7]- [11], system (1) is assumed to be stabilizable.

The performance index of each non-competitive player $i \in \mathcal{M}$ is defined as

$$J_i = \int_0^{\infty} \left( x^T Q_i x + \sum_{j \in \mathcal{M}} u_j^T R_{ij}^{uu} u_j + \sum_{l \in \mathcal{L}} w_l^T R_{il}^{wu} w_l \right) d\tau \tag{2}$$

where matrices $Q_i \in \mathbb{R}^{n\times n}$, $R_{ij}^{uu} \in \mathbb{R}^{m_j\times m_j}$, and $R_{il}^{wu} \in \mathbb{R}^{m_l\times m_l}$ , are symmetric positive definite weighting matrices. The competitive players, for all $l \in \mathcal{L}$, share the following performance index:

$$J_T = \int_0^{\infty} \left( x^T Q_T x + \sum_{l \in \mathcal{L}} \sum_{j \in \mathcal{M}} u_j^T R_{lj}^{uw} u_j + \sum_{l \in \mathcal{L}} w_l^T R_{ll}^{ww} w_l - \gamma^2 \|d\|^2 \right) d\tau, \tag{3}$$

with symmetric positive definite matrices $Q_T \in \mathbb{R}^{n\times n}$, $R_{lj}^{uw} \in \mathbb{R}^{m_j\times m_j}$, $R_{ll}^{ww} \in \mathbb{R}^{m_l\times m_l}$ and the positive constant $\gamma$ denoting disturbance attenuation level.

**Remark 1.** Let $w = [w_1^T, \dots, w_L^T]^T$ denote the stacked vector of competitive players with block diagonal weighting matrix $R^{ww} = \mathrm{diag}(R_{11}^{ww}, \dots, R_{LL}^{ww})$ . Under this representation, standard Nash equilibrium definitions and PI algorithms in [7], [8], and [10] can be directly applied, while the individual optimal competitive policies are reconstructed from the aggregated solution. Consequently, the explicit multiplayer representation of the competitive component is leveraged to design distinct local triggering rules for individual competitive control channels within the proposed MET control analysis.

**Remark 2.** Similar to [17], in this work the adversarial disturbance is considered continuous-time.

The value functions associated with non-competitive players are given by

$$V_i^* = \min_{u_i} \int_t^{\infty} \left( x^T Q_i x + \sum_{j \in \mathcal{M}} u_j^T R_{ij}^{uu} u_j + \sum_{l \in \mathcal{L}} w_l^T R_{il}^{wu} w_l \right) d\tau = x^T P_i x, \tag{4}$$

where $P_i \in \mathbb{R}^{n\times n}$ , for all $i \in \mathcal{M}$ are positive definite matrices. The competitive players share the following value function

$$V_T^* = \min_{w} \max_{d} \int_t^{\infty} \left( x^T Q_T x + \sum_{l \in \mathcal{L}} \sum_{j \in \mathcal{M}} u_j^T R_{lj}^{uw} u_j + \sum_{l \in \mathcal{L}} w_l^T R_{ll}^{ww} w_l - \gamma^2 \|d\|^2 \right) d\tau = x^T P_T x, \tag{5}$$

where $P_T \in \mathbb{R}^{n\times n}$ is positive definite.

**Definition 1.** *(MZS game): The non-competitive players* $\{u_1, \dots, u_M\}$ *and competitive group* $\{w\}$ *collectively constitute the NZS part while* $\{w\}$ *and* $\{d\}$ *form the ZS component of the game.*

**Definition 2.** *(Nash equilibrium): Following [7] and [10], the Nash equilibrium solution of the MZS game is defined as*

$$\begin{aligned} &V_i^*(u_1^*, \dots, u_i^*, \dots, u_M^*, w^*) \\ &\le V_i(u_1^*, \dots, u_i, \dots, u_M^*, w^*), \forall i \in \mathcal{M} \\ &V_T^*(u_1^*, \dots, u_i^*, \dots, u_M^*, w^*) \\ &\le V_T(u_1^*, \dots, u_i^*, \dots, u_M^*, w), \end{aligned} \tag{6}$$

*while the pair* $\{w^*, d^*\}$ *satisfies*

$$V_T(w^*, d) \le V_T^*(w^*, d^*) \le V_T(w, d^*). \tag{7}$$

Employing above definitions, the associated Hamiltonians of players will have the following forms:

$$H_i = x^T Q_i x + \sum_{j \in \mathcal{M}} u_j^T R_{ij}^{uu} u_j + \sum_{l \in \mathcal{L}} w_l^T R_{il}^{wu} w_l + \nabla V_i^T \left( Ax + \sum_{j \in \mathcal{M}} B_j\, u_j + \sum_{l \in \mathcal{L}} C_l\, w_l + E\, d \right) \tag{8}$$

and

$$H_T = x^T Q_T x + \sum_{l \in \mathcal{L}} \sum_{j \in \mathcal{M}} u_j^T R_{lj}^{uw} u_j + \sum_{l \in \mathcal{L}} w_l^T R_{ll}^{ww} w_l + \nabla V_T^T \left( Ax + \sum_{j \in \mathcal{M}} B_j\, u_j + \sum_{l \in \mathcal{L}} C_l\, w_l + E\, d \right) - \gamma^2 \|d\|^2, \tag{9}$$

where $\nabla V_i \triangleq \partial V_i / \partial x$ and $\nabla V_T \triangleq \partial V_T / \partial x$ and we have $H_i^* = H_T^* = 0$ . Employing stationarity and Isaac conditions [8] with $\partial H_i^* / \partial u_i^* = 0$ and $\partial H_T^* / \partial w^* = 0$, the optimal policies for non-competitive players are derived as

$$u_i^* = -R_{ii}^{uu^{-1}} B_i^T P_i\, x = -K_i^u\, x, \quad \forall i \in \mathcal{M}, \tag{10}$$

where $K_i^u$ denotes the state-feedback gain associated with controller $u_i$. The optimal policy for competitive group can be also cast as $w^* = -R^{ww^{-1}} C^T P_T\, x = -K^w\, x$ . Since $R^{ww}$ is block diagonal, the optimal control law for each competitive player can be recovered individually as

$$w_l^* = -R_{ll}^{ww^{-1}} C_l^T P_T\, x = -K_l^w\, x, \ \ \forall l \in \mathcal{L}, \tag{11}$$

with $K_l^w$ denoting the state-feedback gain associated with controller $w_l$. The worst disturbance (see [8]) w.r.t (5) is also obtained employing $\partial H_T^* / \partial d^* = 0$ by

$$d^* = \gamma^{-2} E^T P_T\, x = K_d\, x. \tag{12}$$

Substituting optimal control laws (10), (11), and (12) in (4) and (5), yields the $\{M+1\}$-coupled GARE, where iterative methods such as PI algorithm [7] can be used to find the optimal solutions by finding $P_i$ for all $i \in \mathcal{M}$ and $P_T$. It should be noted that the disturbance considered in this work represents a maximizing adversarial player in the MZS game formulation rather than an unknown exogenous disturbance. The adversarial strategy is obtained through the HJI-based optimization and is explicitly incorporated into the triggering and stability analysis in the subsequent sections. The proposed approach assumes known system dynamics, therefore, robustness against parametric uncertainties and unmodeled dynamics is not considered in this work and remains an open direction for future research.

In the following sections we provide MET control design and analysis using Nash solutions above and optimal control policies (10), (11), and (12).

## 3. MET Control and Global Triggering Rule

In this section, we first extend the MET control formulation used in [20] to the mixed-zero sum game of (1), and then we derive a stability condition and a global triggering rule for players involved in the game.

### *3.1. MET Control Architecture of MZS games*

In most event-triggered control schemes, a measurement error signal is defined as the gap between the current state and the latest sampled state. This error signal is continuously monitored and once it exceeds a pre-defined triggering condition (TC), the new state is sampled and the controllers update their values accordingly and are held constant during inter-event times. In other words, $u(t) = u(x_k)$ for all $t \in [t_k, t_{k+1})$, where $x_k$ is the sampled state at $t_k$ and $t_{k+1}$ denotes the subsequent triggering time. The method has been successfully extended to NZS games and MZS games. However, this centralized triggering method requires synchronous triggering instants.

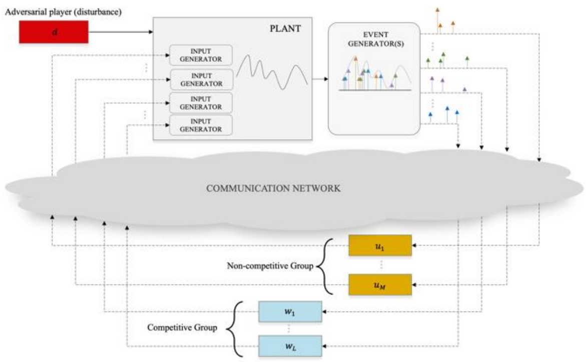



**Fig. 1.** MET control architecture for MZS games

Contrary to the centralized ET, in MET control framework controllers update according to their individual triggering rules. The decentralized structure of the MET control scheme allows asynchronous controller updates and signal transmission. Here, we extend the MET control method [20] to the MZS game (Fig. 1). Extending the definitions in [20], the triggering instants of the proposed MET control framework for MZS game of (1) can be defined.

**Definition 3.** *(Triggering instants): The set of global triggering instants is defined by* $\Theta \triangleq \{\theta_j\}_{j\in\mathcal{M}} \cup \{\theta_l\}_{l\in\mathcal{L}}$, *where* $\theta_j = \{t^j_{k_j}\}_{k_j\in\mathbb{N}}$ *and* $\theta_l = \{t^l_{k_l}\}_{k_l\in\mathbb{N}}$ *denote the sets of sampling times for non-competitive and competitive players, respectively. The* $k_j$*-th triggering instant of non-competitive player* $j \in \mathcal{M}$ *is denoted by* $t^j_{k_j}$ *and the* $k_l$*-th triggering instant of competitive player* $l \in \mathcal{L}$ *is denoted by* $t^l_{k_l}$. *WLOG,* $t^j_{k_j}$ *and* $t^l_{k_l}$ *are used to denote the most recent sampling instants of player* $j \in \mathcal{M}$ *and player* $l \in \mathcal{L}$.

Using above definition, the MET control policies are cast as

$$\overline{u^*}_i = u_i^*\left(t^i_{k_i}\right),\ t \in \left[t^i_{k_i},\ t^i_{k_i+1}\right) \tag{13}$$

$$\overline{w^*}_l = w_l^*\left(t^l_{k_l}\right),\ t \in \left[t^l_{k_l},\ t^l_{k_l+1}\right). \tag{14}$$

The distinct measurement errors for non-competitive players and team players can be then defined by

$$e_i^u \triangleq x^i_{k_i} - x, \forall i \in \mathcal{M} \tag{15}$$

$$e_l^w \triangleq x^l_{k_l} - x, \forall l \in \mathcal{L}, \tag{16}$$

where $x^i_{k_i} \triangleq x(t^i_{k_i})$ and $x^l_{k_l} \triangleq x(t^l_{k_l})$. Next, we define two groups of collective measurement errors. One capturing the collective measurement error of non-competitive players while the other one encapsulates the collective measurement error of competitive players:

$$\boldsymbol{e_g^u} \triangleq \left[e_1^{u\,T}, \quad \cdots \quad , e_M^{u\,T}\right]^T \tag{17}$$

$$\boldsymbol{e_g^w} \triangleq \left[e_1^{w\,T}, \quad \cdots \quad , e_L^{w\,T}\right]^T. \tag{18}$$

Consistent with event-triggered control methods [13]–[20], it is assumed that the full system state is available for evaluating the triggering conditions. In practice, this assumption requires access to state information at the event generators. This assumption allows the focus to remain on the design and stability analysis of the event-triggering mechanism in the subsequent sections. Extending the proposed framework to output-feedback implementations through suitable state observers or estimators is an interesting direction for future research.

### *3.2. Stability Condition under global TC*

Employing MET control policies (13) and (14), the resulting closed-loop system contains both flow dynamics when no player is required to update and also jump dynamics when a subset of players must update their values. The following theorem provides stability guarantees under a global triggering rule for the resulting impulsive system, capturing both flow dynamics during global IETs (i.e. $t \notin \Theta$) and at subsequent triggering instants when $t \in \Theta$.

**Theorem 1**. Consider the MZS game of linear system (1), where the non-competitive players have value functions given by (4) and the competitive players jointly attenuate the continuous-time disturbance while sharing the value function in (5). Let the design parameter $\beta \in (0,1)$, then the closed-loop system under MET control policies (13) and (14), with group measurement errors (17) and (18), and the disturbance law (12) is exponentially stable if $\mu \succ 0$ and the following global triggering condition holds:

$$\boldsymbol{e_g^{u\,T}}\Psi^u\boldsymbol{e_g^u} + \boldsymbol{e_g^{w\,T}}\Psi^w\boldsymbol{e_g^w} \le \beta\lambda_{min}(\mu)\|x\|^2, \tag{19}$$

where $\Psi^u = \frac{1}{\lambda}\left(L^{u^T}L^u\right)$, $\Psi^w = \frac{1}{\lambda}\left(L^{w^T}L^w\right)$ and with

$$\mu = Q_\Sigma + F^u + F^w - \gamma^{-2}\Gamma - 2\lambda I_n, \tag{20}$$

$$F^u = K^{u^T}R^uK^u, \qquad F^w = K^{w^T}R^wK^w,$$

$$\Gamma = P_TEE^TP_T, \qquad Q_\Sigma = \sum_{i\in\mathcal{M}} Q_i + Q_T,$$

$$M_i^u = P_i^{-1}K_i^{u^T}R_{ii}^{uu}K_i^u, \qquad M_l^w = P_T^{-1}K_l^{w^T}R_{ll}^{ww}K_l^w,$$

$$R^u = diag\left(\sum_{i\in\mathcal{M}} R_{i1}^{uu} + \sum_{l\in\mathcal{L}} R_{l1}^{uw}, \dots, \sum_{i\in\mathcal{M}} R_{iM}^{uu} + \sum_{l\in\mathcal{L}} R_{lM}^{uw}\right),$$

$$R^w = diag\left(\sum_{i\in\mathcal{M}} R_{i1}^{wu} + R_{11}^{ww}, \ldots, \sum_{i\in\mathcal{M}} R_{iL}^{wu} + R_{LL}^{ww}\right),$$
$$L^u = \left[\left(\sum_{i\in\mathcal{M}} P_i + P_T\right) M_1^u, \ldots, \left(\sum_{i\in\mathcal{M}} P_i + P_T\right) M_M^u\right],$$
$$L^w = \left[\left(\sum_{i\in\mathcal{M}} P_i + P_T\right) M_1^w, \ldots, \left(\sum_{i\in\mathcal{M}} P_i + P_T\right) M_L^w\right].$$

**Proof** (of Theorem 1). Consider the following candidate Lyapunov function:

$$L = \sum_{i\in\mathcal{M}} V_i^* + V_T^* = \sum_{i\in\mathcal{M}} x^T P_i x + x^T P_T x, \tag{21}$$

with $V_i^*$, $V_T^*$, $P_i$, and $P_T$ defined in (4) and (5). The Lyapunov analysis is then carried over two cases: when $t \notin \Theta$ (global IET) and when $t \in \Theta$ (at consequent triggering instant).

**Case 1:** derivative of $L$ along system solutions will be:

$$\dot{L} = \sum_{i\in\mathcal{M}} \dot{V}_i^* + \dot{V}_T^* \tag{22}$$
$$= \left(\sum_{i\in\mathcal{M}} \nabla V_i^{*T} + \nabla V_T^{*T}\right)\left(Ax + \sum_{j\in\mathcal{M}} B_j\, \overline{u^*}_j + \sum_{l\in\mathcal{L}} C_l\, \overline{w^*}_l + E\, d^*\right).$$

Employing the Hamiltonians (8) and (9), the terms $\nabla V_i^{*T} Ax$ and $\nabla V_T^{*T} Ax$ can be replaced and the derivative of $L$ in (22) can be rewritten as:

$$\begin{aligned}\dot{L} = & -x^T\left(\sum_{i\in\mathcal{M}} Q_i + Q_T\right)x - \sum_{i\in\mathcal{M}}\sum_{j\in\mathcal{M}} u_j^{*T} R_{ij}^{uu} u_j^* \\ & - \sum_{l\in\mathcal{L}}\sum_{j\in\mathcal{M}} u_j^{*T} R_{lj}^{uw} u_j^* \\ & - \sum_{i\in\mathcal{M}}\sum_{l\in\mathcal{L}} w_l^{*T} R_{il}^{wu} w_l^* \\ & - \sum_{l\in\mathcal{L}} w_l^{*T} R_{ll}^{ww} w_l^* \\ & + \sum_{i\in\mathcal{M}} \nabla V_i^{*T} \sum_{j\in\mathcal{M}} B_j(\overline{u^*}_j - u_j^*) \\ & + \sum_{i\in\mathcal{M}} \nabla V_i^{*T} \sum_{l\in\mathcal{L}} C_l(\overline{w^*}_l - w_l^*) \\ & + \gamma^2\|d^*\|^2 \\ & + \nabla V_T^{*T} \sum_{j\in\mathcal{M}} B_j(\overline{u^*}_j - u_j^*) \\ & + \nabla V_T^{*T} \sum_{l\in\mathcal{L}} C_l(\overline{w^*}_l - w_l^*).\end{aligned} \tag{23}$$

Using augmented block matrices $R^u$ and $R^w$ in (20) and employing block gain matrices $K^u = [K_i^{u^T}, \ldots, K_M^{u^T}]^T$ and $K^w = [K_1^{w^T}, \ldots, K_L^{w^T}]^T$, and block matrix $Q_\Sigma = \sum_{i\in\mathcal{M}} Q_i + Q_T$, the derivative of the candidate Lyapunov function becomes

$$\begin{aligned}\dot{L} = & -x^T Q_\Sigma x - x^T K^{u^T} R^u K^u\, x + \gamma^2\|d^*\|^2 \\ & -x^T K^{w^T} R^w K^w x - 2x^T \sum_{i\in\mathcal{M}} P_i \sum_{j\in\mathcal{M}} B_j K_j^u e_j^u \\ & -2x^T \sum_{i\in\mathcal{M}} P_i \sum_{l\in\mathcal{L}} C_l K_l^w e_l^w - 2x^T P_T \sum_{j\in\mathcal{M}} B_j K_j^u e_j^u \\ & -2x^T P_T \sum_{l\in\mathcal{L}} C_l K_l^w e_l^w.\end{aligned} \tag{24}$$

Introducing identity matrices of the form $P_j^{-1}P_j$, $R_{jj}^{uu^{-1}} R_{jj}^{uu}$, $P_T^{-1}P_T$, and $R_{ll}^{ww^{-1}} R_{ll}^{ww}$ to the cross-coupling terms including the measurement errors and the state, the above expression is rewritten as

$$\begin{aligned}\dot{L} = & -x^T Q_\Sigma x - x^T K^{u^T} R^u K^u\, x - x^T K^{w^T} R^w K^w x \\ & - 2x^T \sum_{i\in\mathcal{M}} P_i \sum_{j\in\mathcal{M}} (P_j^{-1}P_j) B_j (R_{jj}^{uu^{-1}} R_{jj}^{uu}) K_j^u e_j^u \\ & - 2x^T \sum_{i\in\mathcal{M}} P_i \sum_{l\in\mathcal{L}} (P_T^{-1}P_T) C_l (R_{ll}^{ww^{-1}} R_{ll}^{ww}) K_l^w e_l^w \\ & - 2x^T P_T \sum_{j\in\mathcal{M}} (P_j^{-1}P_j) B_j (R_{jj}^{uu^{-1}} R_{jj}^{uu}) K_j^u e_j^u \\ & - 2x^T P_T \sum_{l\in\mathcal{L}} (P_T^{-1}P_T) C_l (R_{ll}^{ww^{-1}} R_{ll}^{ww}) K_l^w e_l^w \\ & + \gamma^2\|d^*\|^2 .\end{aligned} \tag{25}$$

Replacing $R_{jj}^{uu^{-1}} B_j^T P_j$ and $R_{ll}^{ww^{-1}} B_l^T P_l$ with $K_j^u$ and $K_l^w$, (27) is rearranged to

$$\begin{aligned}\dot{L} = & -x^T Q_\Sigma x - x^T F^u\, x - x^T F^w x + \gamma^2\|d^*\|^2 \\ & -2x^T \sum_{i\in\mathcal{M}} P_i \sum_{j\in\mathcal{M}} M_j^u e_j^u - 2x^T P_T \sum_{j\in\mathcal{M}} M_j^u e_j^u \\ & -2x^T \sum_{i\in\mathcal{M}} P_i \sum_{l\in\mathcal{L}} M_l^w e_l^w - 2x^T P_T \sum_{l\in\mathcal{L}} M_l^w e_l^w,\end{aligned} \tag{26}$$

with $F^u$, $F^w$, $M_j^u$ and $M_l^w$ defined in (20). Therefore

$$\dot{L} = -x^T Q_\Sigma x - x^T F^u\, x - x^T F^w x - 2x^T L^u \boldsymbol{e_g^u} - 2x^T L^w \boldsymbol{e_g^w} + \gamma^2\|d^*\|^2, \tag{27}$$

where group measurement errors $\mathbf{e_g^u}$ and $\mathbf{e_g^w}$ are used and block matrices $L^u$ and $L^w$ are defined in (20). Using Young's inequality to bound the cross terms we have

$$-2x^T L^u \boldsymbol{e_g^u} \le c_1\, x^T x + \tfrac{1}{c_1} \boldsymbol{e_g^u}^T L^{u^T} L^u \boldsymbol{e_g^u} \tag{28}$$
$$-2x^T L^w \boldsymbol{e_g^w} \le c_2\, x^T x + \tfrac{1}{c_2} \boldsymbol{e_g^w}^T L^{w^T} L^w \boldsymbol{e_g^w}, \tag{29}$$

for any positive scalars $c_1$ and $c_2$. Incorporating bounds in (30) and (31) and setting $c_1 = c_2 = \lambda > 0$, $\dot{L}$ in (29) can be supressed by

$$\begin{aligned}\dot{L} \le & -x^T(Q_\Sigma + F^u + F^w - \gamma^{-2}\Gamma - 2\lambda I_n)x \\ & + \boldsymbol{e_g^u}^T \left(\tfrac{1}{\lambda} L^{u^T} L^u\right) \boldsymbol{e_g^u} + \boldsymbol{e_g^w}^T \left(\tfrac{1}{\lambda} L^{w^T} L^w\right) \boldsymbol{e_g^w},\end{aligned} \tag{30}$$

where we have used $d^* = \gamma^{-2} E^T P_T\, x$ from (12) and $\Gamma = P_T E E^T P_T$ from (20). Equivalently, by using $\mu$ from (20), inequality (32) can be compactly rewritten as

$$\dot{L} \le -x^T \mu x + \boldsymbol{e_g^u}^T \left(\tfrac{1}{\lambda} L^{u^T} L^u\right) \boldsymbol{e_g^u} + \boldsymbol{e_g^w}^T \left(\tfrac{1}{\lambda} L^{w^T} L^w\right) \boldsymbol{e_g^w}. \tag{31}$$

Using Rayleigh's quotients, the term $-x^T \mu x$ can be bounded by $-\lambda_{min}(\mu)\|x\|^2$ provided that $\mu$ is positive definite. Therefore, (33) can be further supressed as follows:

$$\dot{L} \le -\lambda_{min}(\mu)\|x\|^2 + \boldsymbol{e_g^u}^T \left(\tfrac{1}{\lambda} L^{u^T} L^u\right) \boldsymbol{e_g^u} + \boldsymbol{e_g^w}^T \left(\tfrac{1}{\lambda} L^{w^T} L^w\right) \boldsymbol{e_g^w}. \tag{32}$$

Therefore, incorporating global triggering condition (19) with design parameter $\beta \in (0,1)$ guarantees exponential stability of the closed-loop during IETs.

**Case 2**. From (21) we have $\lambda_{min}(\sum_{i\in\mathcal{M}} P_i + P_T)\|x\|^2 \leq L \leq \lambda_{max}(\sum_{i\in\mathcal{M}} P_i + P_T)\|x\|^2$ while also under global triggering condition (19), from (34) we have $\dot{L} \leq -(1-\beta)\lambda_{min}(\mu)\|x\|^2$. Therefore, the decay of $L$ is bounded exponentially under the proposed global MET conditions of Theorem 1. Despite the jumps at control policies, the continuity of $x(t)$ is preserved at any given time, i.e., $x(t^+) = x(t^-)$. Therefore, the continuity of $L$ is also preserved at subsequent sampling instant or equivalently

$$L(t^+) = L(t^-). \tag{33}$$

The stability of the closed-loop during IETs and the exponential decay of $L$ also guarantees

$$L({t_{k+1}}^-) < L({t_k}^+), \tag{34}$$

where $t_k$ is used to denote the previous triggering instant and $t_{k+1}$ is used to denote the consequent one. Therefore, (33) and (34) imply $L(t_{k+1}) < L(t_k)$ and the Lyapunov function monotonically decreases across successive triggering instants which completes the proof. ■

To further analyze the feasibility of the derived triggering condition, the following proposition provides an equivalent LMI characterization of the feasibility condition in Theorem 1.

**Proposition 1**. The condition $\mu \succ 0$ from Theorem 1 with $\mu$ defined in (20), is equivalently expressed by the following LMI:

$$\begin{bmatrix} Q_\Sigma + F^u + F^w - 2\lambda I_n & P_T E \\ E^T P_T & \gamma^2 I_n \end{bmatrix} \succ 0 \tag{35}$$

**Proof:** The proof directly follows from application of the Schur complement. ■

**Remark 3:** Proposition 1 provides an equivalent LMI characterization of the condition $\mu \succ 0$, which facilitates its verification in terms of the weighting matrices and the GARE solution $P_T$. Furthermore, by Weyl's inequality,

$$\lambda_{min}(\mu) \geq \lambda_{min}(Q_\Sigma + F^u + F^w - 2\lambda I_n) - \gamma^{-2}\lambda_{max}(\Gamma).$$

Since $\lambda_{min}(Q_\Sigma + F^u + F^w - 2\lambda I_n)$ is equivalent to $\lambda_{min}(Q_\Sigma + F^u + F^w) - 2\lambda$, the condition $\mu \succ 0$ is guaranteed if there exists a positive Young's parameter $\lambda$ satisfying

$$0 < \lambda < \frac{\lambda_{min}(Q_\Sigma + F^u + F^w) - \gamma^{-2}\lambda_{max}(\Gamma)}{2}. \tag{36}$$

The above results provide theoretical foundation to design decentralized triggering rules for the MET controllers as discussed in the subsequent section.

## 4. Local Budgets and Local TCs

Although Theorem 1 provides a global triggering condition, it does not directly yield decentralized triggering rules since it depends on the aggregate error functionals. In this section, it is first shown that deriving local triggering conditions for the MET controllers reduces to designing distinct triggering thresholds under sufficient local triggering conditions. Then, leveraging the provided design flexibility, a hierarchical allocation mechanism is developed, enabling to tailor these local rules according to the application and specific design requirements. Finally, computational burden of the proposed decentralized triggering mechanism for offline synthesis and during online implementation is discussed.

### *4.1. Sufficient conditions under local TCs*

The following theorem establishes sufficient local triggering conditions by introducing local budget-allocation weights that distributes the admissible global triggering budget among the individual competitive and non-competitive players.

**Theorem 2.** The global triggering condition (19) is guaranteed if for any non-competitive player $i \in \mathcal{M}$ and for any competitive player $l \in \mathcal{L}$ the following inequalities hold:

$$\|e_i^u\|^2 \leq \varpi_i^u \left(\frac{\beta\, \lambda_{min}(\mu)\|x\|^2}{\lambda_{max}(\Psi_i^u)}\right), \tag{37a}$$

$$\|e_l^w\|^2 \leq \varpi_l^w \left(\frac{\beta\, \lambda_{min}(\mu)\|x\|^2}{\lambda_{max}(\Psi_l^w)}\right), \tag{37b}$$

where $\Psi_i^u = \psi_{ii}^u + \sum_{j\neq i}\|\psi_{ij}^u\| I_n$ and $\Psi_l^w = \psi_{ll}^w + \sum_{k\neq l}\|\psi_{lk}^w\| I_n$, while $\varpi_i^u \in (0,1)$ and $\varpi_l^w \in (0,1)$ denote local budget-allocation weights, satisfying the following distribution constraint:

$$\sum_{i\in\mathcal{M}} \varpi_i^u + \sum_{l\in\mathcal{L}} \varpi_l^w = 1. \tag{38}$$

**Proof** (of Theorem2). Global triggering condition (19) can be equivalently expressed by

$$\boldsymbol{e_g^u}^{\boldsymbol{T}} \Psi^u \boldsymbol{e_g^u} + \boldsymbol{e_g^w}^{\boldsymbol{T}} \Psi^w \boldsymbol{e_g^w} \leq \boldsymbol{\Phi_g}, \tag{39}$$

where $\boldsymbol{\Phi_g} = \beta\lambda_{min}(\mu)\|x\|^2$. The matrix $\Psi^u$ is symmetric, therefore the group error functionals $\mathbf{e_g^u}^{\boldsymbol{T}}\Psi^u\mathbf{e_g^u}$ can be expanded in terms of diagonal and off-diagonal entries as follows:

$$\boldsymbol{e_g^u}^{\boldsymbol{T}} \Psi^u \boldsymbol{e_g^u} \leq \sum_{i\in\mathcal{M}} {e_i^u}^T \psi_{ii}^u e_i^u + \sum_{i\in\mathcal{M}}\sum_{j\neq i} {e_i^u}^T \psi_{ij}^u e_j^u \tag{40}$$

Employing Cauchy-Schwartz inequality, (40) can be upper bounded by

$$\boldsymbol{e_g^u}^{\boldsymbol{T}} \Psi^u \boldsymbol{e_g^u} \leq \sum_{i\in\mathcal{M}} {e_i^u}^T \psi_{ii}^u e_i^u + \sum_{i\in\mathcal{M}}\sum_{j\neq i} \|\psi_{ij}^u\|\|e_i^u\|\|e_j^u\| \tag{41}$$

where $\|\psi_{ij}^u\|$ is the spectral norm of $\psi_{ij}^u$. Applying Young's inequality further yields

$$\boldsymbol{e_g^u}^{\boldsymbol{T}} \Psi^u \boldsymbol{e_g^u} \leq \sum_{i\in\mathcal{M}} {e_i^u}^T \psi_{ii}^u e_i^u + \frac{1}{2}\sum_{i\in\mathcal{M}}\sum_{j\neq i} \|\psi_{ij}^u\|\|e_i^u\|^2 + \frac{1}{2}\sum_{i\in\mathcal{M}}\sum_{j\neq i} \|\psi_{ij}^u\|\|e_j^u\|^2, \tag{42}$$

Employing index change for the last term, while using symmetry ${\psi_{ij}^u}^T = \psi_{ji}^u$, (42) can be rearranged as follows

$$\boldsymbol{e_g^{u^T}} \Psi^u \boldsymbol{e_g^u} \leq \sum_{i \in \mathcal{M}} e_i^{u^T} \left( \psi_{ii}^u + \sum_{j \neq i} \| \psi_{ij}^u \| I_n \right) e_i^u , \quad (43)$$

Similarly, the group functional of the competitive players admit the following upper bound:

$$\boldsymbol{e_g^{w^T}} \Psi^w \boldsymbol{e_g^w} \leq \sum_{l \in \mathcal{L}} e_l^{w^T} \left( \psi_{ll}^w + \sum_{k \neq l} \| \psi_{lk}^w \| I_n \right) e_l^w \quad (44)$$

with $k \in \mathcal{L}$. Employing Rayleigh quotients, each term in the RHS of inequalities (43) and (44) can be further upper bounded by $\lambda_{max}(\Psi_i^u)\|e_i^u\|^2$ and $\lambda_{max}(\Psi_l^w)\|e_l^w\|^2$, respectively. Therefore, a sufficient condition for (39) is given by

$$\begin{aligned} \sum_{i \in \mathcal{M}} \lambda_{max}(\Psi_i^u)\|e_i^u\|^2 + \sum_{l \in \mathcal{L}} \lambda_{max}(\Psi_l^w)\|e_l^w\|^2 & \\ \leq \boldsymbol{\Phi_g} & \\ = \left( \sum_{i \in \mathcal{M}} \varpi_i^u \right. & \quad (45) \\ \left. + \sum_{i \in \mathcal{L}} \varpi_i^w \right) \boldsymbol{\Phi_g} , & \end{aligned}$$

where the last equality follows from (38). Consequently, (45) is satisfied, if the following inequalities hold:

$$\|e_i^u\|^2 \leq \varpi_i^u \left( \frac{\boldsymbol{\Phi_g}}{\lambda_{max}(\Psi_i^u)} \right), \quad (46a)$$

$$\|e_l^w\|^2 \leq \varpi_l^w \left( \frac{\boldsymbol{\Phi_g}}{\lambda_{max}(\Psi_l^w)} \right). \quad (46b)$$

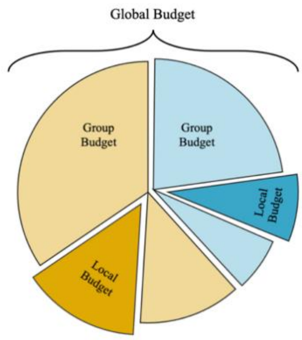
Hence, if the distribution constraint (38) is satisfied, then the local triggering rules (37) satisfy (45), which in turn guarantees the global triggering condition (19). ■

**Remark 4:** Without loss of generality, only active players with nonzero feedback policies $K_i^u$ and $K_l^w$ defined in (10) and (11), are considered in the game formulation. If a player has no influence on the system dynamics through its input channel, the corresponding player can be excluded and the game can be reformulated with the remaining players. For the considered active players, the GARE feedback policies yield nonzero contributions to the corresponding blocks of $L^u$ and $L^w$. Since $\Psi^u = \frac{1}{\lambda}\left(L^{u^T} L^u\right)$ and $\Psi^w = \frac{1}{\lambda}\left(L^{w^T} L^w\right)$, the corresponding diagonal blocks $\psi_{ii}^u$ and $\psi_{ll}^w$ are nonzero semidefinite matrices. Therefore, local matrices $\Psi_i^u$ and $\Psi_l^w$ in (37) are also nonzero and positive semidefinite, which guarantees $\lambda_{max}(\Psi_i^u) > 0$ and $\lambda_{max}(\Psi_l^w) > 0$, ensuring that the local triggering thresholds in (37) are well-defined.

**Remark 5:** It is worth noting that the triggering conditions are derived as sufficient conditions and may therefore introduce conservativeness due to the employed bounding operations. The use of player-dependent bounds in Theorem~2 provides less restrictive local triggering thresholds compared with using common bounds for all controllers.

Theorem 2 establishes closed-loop stability under local triggering rules; however, it does not uniquely dictate how the admissible global triggering budget should be distributed among individual players. Consequently, any collection of budget-allocation weights satisfying (38) yields a valid distribution and preserves the local and global triggering conditions established in Theorems 1 and 2. This inherent design flexibility allows the budget-allocation weights to be selected according to the design preferences and requirements of the application under consideration. The following subsection introduces a hierarchical allocation mechanism, providing a structured parameterization of the admissible budget-allocation weights.

### *4.2. Hierarchical allocation mechanism*




**Step 1:** Global-to-group allocation
**Step 2:** Group-to-local allocation

**Fig. 2.** Hierarchical allocation.

In this subsection, the admissible budget-allocation weights introduced in Theorem 2 are constructed through a hierarchical allocation mechanism. The global triggering budget is first partitioned between the non-competitive and competitive groups and is subsequently distributed among the individual players within each group. Within such structured parameterization, particular solutions are suggested as illustrative design examples. The hierarchical allocation procedure can be carried over two steps represented in Fig. 2.

**Step 1:** We determine the triggering thresholds of non-competitive and competitive groups by assigning global-to-group coefficients as follows:

$$\boldsymbol{e_g^{u^T}} \Psi^u \boldsymbol{e_g^u} \leq \sum_{i \in \mathcal{M}} \lambda_{max}(\Psi_i^u)\|e_i^u\|^2 \leq \sigma^u \boldsymbol{\Phi_g} = \boldsymbol{\Phi_g^u} \quad (47)$$

$$\boldsymbol{e_g^{w^T}} \Psi^w \boldsymbol{e_g^w} \leq \sum_{l \in \mathcal{L}} \lambda_{max}(\Psi_l^w)\|e_l^w\|^2 \leq \sigma^w \boldsymbol{\Phi_g} = \boldsymbol{\Phi_g^w} \quad (48)$$

where (47) and (48) are the results of (45) and

$$\sigma^u + \sigma^w = 1. \quad (49)$$

Therefore, $\sigma^u$ can determine what portion of the global budget is assigned to the group of non-competitive players and $\sigma^w$ dictates the share of the competitive group. For instance, assigning a larger share of the global budget to a group, allows more tolerable group measurement error functional while smaller share, leads to smaller threshold for the group. The following example shows how system matrices can be employed to systematically assign these group budgets.

**Example 1 (Global-to-group coefficients):** A particular solution to find the group shares is to employ the overall state weighting matrix $Q_\Sigma$ and the disturbance-related energy matrix $\Gamma$ defined in (20), to assign the global-to-group coefficients:

$$\sigma^u = \frac{\lambda_{max}\,(Q_\Sigma)^{-1}}{\lambda_{max}\,(Q_\Sigma)^{-1} + \lambda_{max}\,(\Gamma)^{-1}}$$
$$\sigma^w = \frac{\lambda_{max}\,(\Gamma)^{-1}}{\lambda_{max}\,(Q_\Sigma)^{-1} + \lambda_{max}\,(\Gamma)^{-1}}. \quad (50)$$

By this particular assignment, if Γ is relatively larger than $Q_\Sigma$, (50) assigns a smaller triggering budget to the competitive group enforcing more frequent updates of the group and enabling to adopt more recent sampled states. Meanwhile, if the disturbance energy is relatively smaller, (50) assigns a larger triggering threshold to the competitive group which allows more tolerable measurement error for the group.

**Step 2:** We assign group-to-local coefficients where each group budget is distributed among players of that group:

$$\lambda_{max}\,(\Psi_i^u)\|e_i^u\|^2 \le \delta_i^u \boldsymbol{\Phi_g^u} \quad (51)$$
$$\lambda_{max}\,(\Psi_l^w)\|e_l^w\|^2 \le \delta_l^w \boldsymbol{\Phi_g^w} \quad (52)$$

where (51) and (52) are obtained from (47) and (48) with

$$\sum_{i\in\mathcal{M}} \delta_i^u = 1 \qquad and \qquad \sum_{l\in\mathcal{L}} \delta_l^w = 1 \quad (53)$$

The allocation of budget from groups to local players indicates which players have been prioritize within the group. For instance, larger local share is equivalent to intentionally allowing more tolerable measurement error for the player, while a relatively smaller coefficient leads to smaller triggering threshold and consequently leads to more frequent updates. Assignment of such coefficients depends on design preferences, application in use or the communication network. The following examples provide particular solutions to assign these coefficients systematically.

**Example 2 (group-to-local coefficients):** Assuming $c_i$ is an indication of the triggering cost for non-competitive player $i$, group-to-local coefficients for non-competitive players can be assigned as

$$\delta_i^u = \frac{c_i}{\sum_{i\in\mathcal{M}} c_i}. \quad (54)$$

If the relative triggering cost of a player is large, a larger local share is assigned to that player, allowing larger measurement errors and fewer updates.

The assignment in (54) can be also incorporated for competitive players. However, the following example represents another particular solution that leverages the MZS system matrices to provide an alternative feasible assignment for such players.

**Example 3 (group-to-local coefficients):** The instantaneous disturbance energy is captured by $\|d^*\|^2 = \gamma^{-4}x^T P_T E E^T P_T x = \gamma^{-4}x^T \Gamma x$ . where Γ can be determined by $P_T E$. Therefore, the projection along $l$-th competitive control channel can be obtained by $S_l^w = C_l^T P_T E$ which can be used to assign group-to-local coefficients for competitive players as follows:

$$\delta_l^w = \frac{1/\|S_l^w\|}{\sum_{l\in\mathcal{L}} 1/\|S_l^w\|} \quad (55)$$

This allocation prioritizes competitive controllers with relatively higher alignment to be assigned by a smaller triggering budget which in turn provides more frequent updates for such players and allows them to adopt more recently sampled states to compete with the adversarial player.

The following corollary establishes stability guarantees when the proposed allocation mechanism is employed.

**Corollary 1.** The hierarchical budget allocation mechanism constitutes an equivalent parameterization of the admissible budget-allocation weights characterized in Theorem 2. Consequently, the corresponding local triggering rules satisfy the global triggering condition of Theorem 1.

**Proof** (of Corollary 1). Employing $\sigma^u$ and $\sigma^w$ from (47) and (48) and $\delta_i^u$ and $\delta_l^w$ from (51) and (52), the budget allocation weights can be constructed by

$$\varpi_i^u = \delta_i^u \sigma^u$$
$$\varpi_l^w = \delta_l^w \sigma^w, \quad (56)$$

therefore, by (49) and (53),

$$\sum_{i\in\mathcal{M}} \varpi_i^u + \sum_{i\in\mathcal{M}} \varpi_l^w = \left(\sum_{i\in\mathcal{M}} \delta_i^u\right)\sigma^u + \underbrace{\left(\sum_{i\in\mathcal{L}} \delta_l^w\right)}_{=1}\sigma^w = \sigma^u + \sigma^w \quad (57)$$

*which satisfies distribution constraint (38). Conversely, let $\varpi_i^u$ and $\varpi_l^w$ be any budget-allocation weights satisfying distribution constraint (38). Then by defining*

$$\sigma^u = \sum_{i\in\mathcal{M}} \varpi_i^u, \qquad \sigma^w = \sum_{i\in\mathcal{M}} \varpi_l^w \quad (58)$$

$$\delta_i^u = \varpi_i^u/\sigma^u, \qquad \delta_l^w = \varpi_l^w/\sigma^w \quad (59)$$

the distribution constraint (38) yields

$$\sigma^u + \sigma^w = \sum_{i\in\mathcal{M}} \varpi_i^u + \sum_{i\in\mathcal{M}} \varpi_l^w = 1$$
$$\sum_{i\in\mathcal{M}} \delta_i^u = \sum_{i\in\mathcal{M}} \varpi_i^u/\sigma^u = \sum_{i\in\mathcal{M}} \sigma^u/\sigma^u = 1 \quad (60)$$
$$\sum_{i\in\mathcal{M}} \delta_l^w = \sum_{i\in\mathcal{M}} \varpi_l^w/\delta_l^w = \sum_{i\in\mathcal{M}} \delta_l^w/\delta_l^w = 1$$

which satisfy conditions (49) and (53). Therefore, every admissible budget allocation satisfying Theorem 2 admits the proposed hierarchical decomposition, and every decomposition coefficient leads to an admissible budget-allocation weight, establishing the equivalence. Consequently, the assumptions of Theorem 2 hold, implying that the corresponding local triggering rules satisfy the global triggering condition (19). ■

### *4.3. Computational burden*

The computational burden of the proposed MET framework can be separated into offline synthesis and online implementation. During the offline synthesis stage, the generalized PI procedure requires solving one Lyapunov equation for each non-competitive player together with one additional Lyapunov equation associated with the competitive players and adversarial player. Consequently, increasing the number of non-competitive players increases the number of Lyapunov equations to be solved, whereas increasing the number of competitive players enlarges the dimensions of the aggregated competitive component while retaining a single competitive value function. In addition, the parameters associated with the local triggering conditions for individual players are determined during this stage. These computations are performed only once during controller synthesis.

During online operation, the event generators utilize the precomputed triggering parameters. The common state-dependent terms in the triggering conditions are evaluated from the current monitored state, while conditions are evaluated independently using local measurement error and its corresponding local triggering threshold. When a triggering condition is satisfied, the associated controller updates its policy using the precomputed feedback gains. Hence, the online implementation does not require solving optimization problems, policy iterations, or Lyapunov equations. Increasing the number of players therefore preserves the decentralized nature of the online computation, where communication and controller updates are triggered only for players satisfying their local triggering conditions.

## 5. Zeno Exclusion

The following theorem establishes strictly positive lower bounds on the inter-event times of the proposed MET controllers to exclude Zeno behavior.

**Theorem 3.** Suppose the MZS game of linear system (1) is equipped with the decentralized MET control laws (13) and (14) with disturbance characterized as (12). Then, under conditions of Theorems 1, 2 and 3, strictly positive lower bounds on IETs for each player $i \in \mathcal{M}$ and for each player $l \in \mathcal{L}$ are guaranteed, given by

$$\begin{aligned} \inf_{k_i}\ \{\tau^i_{k_i}\} &\geq \frac{1}{2\xi} ln\left(1+\min_{k_i}\frac{2\xi\left\|e^u_i\left(t^{i-}_{k_i+1}\right)\right\|}{\eta^u_i}\right) \\ \inf_{k_l}\ \{\tau^l_{k_l}\} &\geq \frac{1}{2\xi} ln\left(1+\min_{k_l}\frac{2\xi\left\|e^w_l\left(t^{l-}_{k_l+1}\right)\right\|}{\eta^w_l}\right), \end{aligned} \tag{70}$$

with $\xi, \eta^u_i, \eta^w_l > 0$.

**Proof** (of Theorem 3). The closed-loop dynamics can be written as

$$\dot{x} = Ax + \sum_{j\in\mathcal{M}} B_j\ \overline{u^*}_j + \sum_{l\in\mathcal{L}} C_l\ \overline{w^*}_l\ + EK_d x. \tag{71}$$

Since input matrices $B_j$ and $C_l$ are constants and control inputs $\overline{u^*}_j$ and $\overline{w^*}_l$ are piecewise constants, we can define the following uniform bounds: $\bar{B} \triangleq \max_j\{\|B_j\|\}$, $\bar{C} \triangleq \max_l\{\|C_l\|\}$, $u_{max} \triangleq \max_j\{\|\overline{u^*}_j\|\}$, and $w_{max} \triangleq \max_l\{\|\overline{w^*}_l\|\}$. Employing these bounds, $\|\dot{x}\|$ admits the following inequality:

$$\|\dot{x}\| \leq \|A\|\|x\| + \|EK_d\|\|x\| + M\ \bar{B}\ u_{max} + L\ \bar{C}\ w_{max}\ , \tag{72}$$

where triangle and Cauchy's inequalities were employed. During IETs of player $i$, the derivative of the state can be replaced by differentiating local measurement errors (15), i.e., $\dot{x} = \dot{x}\left(t^i_{k_i}\right) - \dot{e}^u_i = -\dot{e}^u_i$, since $\dot{x}\left(t^i_{k_i}\right) = 0$. Also setting $\xi \triangleq max\{\|A\|, \|EK_d\|\}$ as the maximum induced 2-norm of matrices $A$ and $EK_d$, for all non-competitive players $i \in \mathcal{M}$, we have

$$\|\dot{e}^u_i\| \leq 2\xi\|e^u_i\| + 2\xi\left\|x\left(t^i_{k_i}\right)\right\| + M\ \bar{B}\ u_{max} + L\ \bar{C}\ w_{max}, \tag{73}$$

where (15) is used to replace $x$ by $x\left(t^i_{k_i}\right) - e^u_i$. Defining $\eta^u_i \triangleq 2\xi\left\|x\left(t^i_{k_i}\right)\right\| + M\bar{B}u_{max} + L\bar{C}w_{max}$, inequality (52) for any non-competitive player $i \in \mathcal{M}$ can be rewritten by

$$\|\dot{e}^u_i\| \leq 2\xi\|e^u_i\| + \eta^u_i. \tag{74}$$

Similarly, during IETs of competitive player $l \in \mathcal{L}$, the derivative of the local measurement error admits the following inequality:

$$\|\dot{e}^w_i\| \leq 2\xi\|e^w_l\| + \eta^w_l, \tag{75}$$

with $\eta^w_l \triangleq 2\xi\left\|x\left(t^l_{k_l}\right)\right\| + M\bar{B}u_{max} + L\bar{C}w_{max}$. The analysis then follows directly by solving differential inequalities (53) and (54) and taking natural logarithm, while assuming the maximum tolerable errors occur prior to violation of the associated TCs at $t^{i-}_{k_i+1}$ and $t^{l-}_{k_l+1}$. Therefore, the remaining steps proceed analogously to [20] and are omitted for brevity. ■

## 6. Simulation results

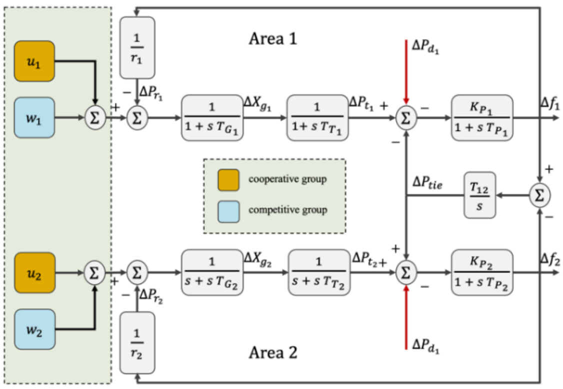



**Fig. 3.** Two-area interconnected power system

This section includes the LFC problem of a two-area interconnected power system to evaluate our proposed MET control approach, while numerical and conceptual comparisons with centralized event-triggered control approaches are also provided. The control objective is to achieve secondary frequency (SF) regulation in both areas in the presence of adversarial disturbances. In the considered setup each area includes one non-competitive and one competitive player in the AGC layer, Fig. 3. The non-competitive and competitive controllers are assumed to share the same actuator channels and the control input applied to each area is the superposition of the control policies of these players. Distinct performance indices are assigned for the non-competitive players, while the competitive players jointly compete with the adversarial player through a shared zero-sum performance index. The system dynamics of each area are adopted from [21] and full state availability is assumed. The system matrices are defined by

$$A = \begin{bmatrix} \frac{-1}{T_{P_1}} & \frac{K_{P_1}}{T_{P_1}} & 0 & 0 & 0 & 0 & \frac{-K_{P_1}}{T_{P_1}} \\ 0 & \frac{-1}{T_{T_1}} & \frac{1}{T_{T_1}} & 0 & 0 & 0 & 0 \\ \frac{-1}{R_1 T_{G_1}} & 0 & \frac{-1}{T_{G_1}} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-1}{T_{P_2}} & \frac{K_{P_2}}{T_{P_2}} & 0 & \frac{K_{P_2}}{T_{P_2}} \\ 0 & 0 & 0 & 0 & \frac{-1}{T_{T_2}} & \frac{1}{T_{T_2}} & 0 \\ 0 & 0 & 0 & \frac{-1}{R_2 T_{G_2}} & 0 & \frac{-1}{T_{G_2}} & 0 \\ T_{12} & 0 & 0 & -T_{12} & 0 & 0 & 0 \end{bmatrix}$$

$B_1 = \left[0,0,\frac{1}{T_{G_1}},0,0,0,0\right]^T$, $B_2 = \left[0,0,0,0,0,\frac{1}{T_{G_2}},0\right]^T$,

$C_1 = \left[0,0,\frac{1}{T_{G_1}},0,0,0,0\right]^T$, $C_2 = \left[0,0,0,0,0,\frac{1}{T_{G_2}},0\right]^T$,

$$E = \begin{bmatrix} \frac{-K_{P_1}}{T_{P_1}} & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{-K_{P_2}}{T_{P_2}} & 0 & 0 & 0 \end{bmatrix}^T$$

Also $x = \left[\Delta f_1, \Delta P_{g_1}, \Delta X_{g_1}, \Delta f_2, \Delta P_{g_2}, \Delta X_{g_2}, \Delta P_{tie}\right]^T$ is the state vector, $d = [\Delta P_{d_1}{}^T, \Delta P_{d_2}{}^T]^T$ is the adversarial player and areas are interconnected via a tie-line. Table I provides system parameters.

**Table I.** Parameters of the two-area power system.

| **area** | $\boldsymbol{T_P}$(s) | $\boldsymbol{T_T}$(s) | $\boldsymbol{T_G}$(s) | $\boldsymbol{K_P}$ | $\boldsymbol{r}$(Hz) | $\boldsymbol{T_{12}}$(p.u.) |
|---|---|---|---|---|---|---|
| **1** | 20 | 0.33 | 0.12 | 121 | 2.4 | 0.545 |
| **2** | 19.5 | 0.27 | 0.08 | 119.5 | 2.4 | 0.545 |

The state weighting matrices are assumed to be $Q_1 = \mathrm{diag}(0.020, 0.006, 0.006, 0.020, 0.006, 0.006, 0.020)$, $Q_2 = 0.5\, Q_1$, and $Q_T = 1.5\, Q_1$. The control weighting matrices are $R_{11}^{uu} = R_{11}^{ww} = 10$, and $R_{22}^{uu} = R_{22}^{ww} = 2.5$ while other cross-coupling weighting matrices are set to zero. The predetermined disturbance attenuation level is set to $\gamma = 4$. The considered LFC system consists of two areas with different physical parameters, while also heterogeneous performance indices are assigned to represent the considered MZS game formulation.
During offline synthesis, the PI algorithm converged at 6th iteration within the required threshold of $10^{-6}$, yielding $P_1$, $P_2$, and $P_T$. Employing Remark 3, the maximum allowable bound for Young's parameter was found at 0.008308 and the parameter was set to $\lambda = 0.00415$. Consequently, the required feasibility condition of Theorem 1 ($\mu \succ 0$) was satisfied with $\lambda_{\min}(\mu) = 0.010625$. The matrices $\Psi_1^u$, $\Psi_1^u$, $\Psi_1^w$, and $\Psi_2^w$, used in local triggering rules (37a) and (37b), computed at this stage while design parameter $\beta$ was set to 0.9. For the online simulation, six different scenarios were incorporated, represented in Table II.

**Table II.** MET control scenarios

| **Scenario** | **global-to-group** $\sigma^u$ | $\sigma^w$ | **group-to-local** $\delta_1^u$ | $\delta_2^u$ | $\delta_1^w$ | $\delta_2^w$ | **budget-allocation weights** $\varpi_1^u$ | $\varpi_2^u$ | $\varpi_1^w$ | $\varpi_2^w$ |
|---|---|---|---|---|---|---|---|---|---|---|
| **1** | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.5 | 0.25 | 0.25 | 0.25 | 0.25 |
| **2** | 0.5 | 0.5 | 0.4 | 0.6 | 0.4 | 0.6 | 0.2 | 0.3 | 0.2 | 0.3 |
| **3** | 0.5 | 0.5 | 0.6 | 0.4 | 0.6 | 0.4 | 0.3 | 0.2 | 0.3 | 0.2 |
| **4** | 0.38858 | 0.61142 | 0.50000 | 0.50000 | 0.56795 | 0.43205 | 0.19429 | 0.19429 | 0.34726 | 0.26416 |
| **5** | 0.38858 | 0.61142 | 0.66667 | 0.33333 | 0.56795 | 0.43205 | 0.25905 | 0.12953 | 0.34726 | 0.26416 |
| **6** | 0.38858 | 0.61142 | 0.33333 | 0.66667 | 0.56795 | 0.43205 | 0.12953 | 0.25905 | 0.34726 | 0.26416 |

These scenarios are selected to evaluate different realizations of the proposed hierarchical procedure to find the admissible local budget-allocation weights ($\varpi_1^u$, $\varpi_2^u$, $\varpi_1^w$, $\varpi_2^w$) required for local triggering rules established in Theorem 2. The required policy updates for players across all simulation scenarios are reported in Table III.
Scenario 1 represents a uniform budget distribution, where global-to-group and group-to-local coefficients of the procedure are assigned equally which yields uniform local budget-allocation weights $\varpi_1^u = \varpi_2^u = \varpi_1^w = \varpi_2^w$. This scenario serves as a baseline case without prioritization. Scenarios 2 and 3 consider asymmetric coefficients to illustrate the effect of redistributing local budgets among players. Compared to Scenario 1 where uniform distribution was utilized, in Scenario 2, smaller group-to-local coefficients were assigned to $u_1$ and $w_1$ and larger coefficients were considered for $u_2$ and $w_2$. Consequently, the local budget-allocation weights $\varpi_1^u$ and $\varpi_1^w$ decreased while $\varpi_2^u$ and $\varpi_2^w$ increased. Therefore, compared to uniform distribution, larger local triggering thresholds are obtained for $u_2$ and $w_2$, allowing more tolerable local measurement errors for these players. Meanwhile smaller triggering budgets are assigned for $u_1$ and $w_1$ imposing more restrictive thresholds and less tolerable local measurement errors. This prioritization has been reflected in Table III, where compared to Scenario 1, $u_2$ and $w_2$ require less policy updates, while $u_1$ and $w_1$ are enforced to update their policies more frequently than non-prioritized uniform distribution. The prioritization in Scenario 3 has been reversed.
Scenarios 4–6 employ the allocation Examples 1-3 in Section 4.2, to assign global-to-group and group-to-local coefficients systematically. The required system matrices for these scenarios were obtained as $\lambda_{\max}(Q_\Sigma) = 0.06$, $\lambda_{\max}(\Gamma) = 0.03813$, $\|S_1^w\| = 0.19150$, and $\|S_2^w\| = 0.25174$. These assignments provide global-to-group coefficients for both groups and group-to-local coefficients for the competitive players. Since theses quantities depend on system matrices, they remain unchanged across these scenarios. However, Scenario 4, assumes equal triggering costs for non-competitive players ( $c_1 = c_2$ ) to provide a baseline case for comparison with asymmetric triggering costs considered in Scenarios 5 and 6, where $c_1 = 2c_2$ and $c_1 = 0.5\, c_2$

were incorporated, respectively. In Scenario 5, by assigning a higher triggering cost for $u_1$ and a lower triggering cost for $u_2$, yields larger budget allocation weight $\varpi_1^u$ and smaller weight $\varpi_2^u$ (Table II), compared to the uniform non-prioritized case of Scenario 4. Consequently, compared to the unform case, a larger triggering threshold is assigned for non-competitive player 1 and a smaller one is assigned for non-competitive player 2. As expected, Table III shows fewer policy updates for $u_1$ and more frequent updates for $u_2$ are required, compared to non-prioritized Scenario 4. The prioritization in Scenario 6 has been reversed.

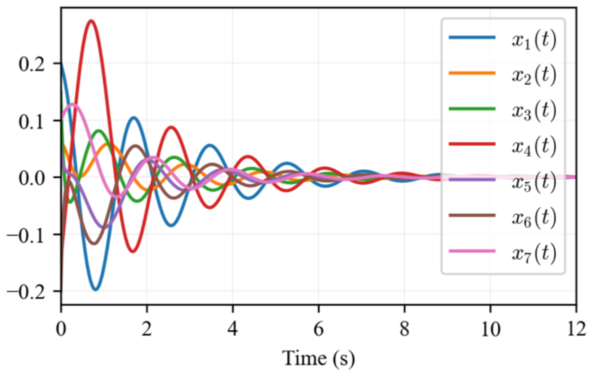



**Fig. 4.** States of the closed-loop system.

**Table III.** Required updates (MET vs TT)

| Method | $u_1$ | $u_2$ | $w_1$ | $w_2$ | Total |
|---|---|---|---|---|---|
| **MET** | | | | | |
| **Scenario 1** | 103 | 123 | 187 | 335 | 748 |
| **Scenario 2** | 115 | 111 | 209 | 306 | 741 |
| **Scenario 3** | 94 | 136 | 171 | 373 | 774 |
| **Scenario 4** | 117 | 138 | 159 | 326 | 740 |
| **Scenario 5** | 101 | 170 | 159 | 326 | 756 |
| **Scenario 6** | 144 | 120 | 159 | 326 | 749 |
| **DETC** | 221 | 221 | 221 | 221 | 884 |
| **SETC** | 673 | 673 | 673 | 673 | 2692 |
| **TT** | 12000 | 12000 | 12000 | 12000 | 48000 |

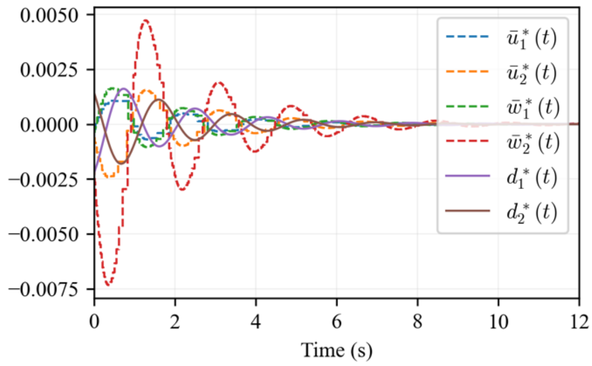



**Fig. 5.** MET controllers and disturbance.

**Table IV.** Comparison of performance metrics (MET scenarios/DETC/SETC)

| Method | IAE $\Delta f_{1,2}$ | IAE *tie* | ITAE $\Delta f_{1,2}$ | $T_s$ $\Delta f_1$ | $T_s$ $\Delta f_2$ | $T_s$ *tie* | $OS$ $\Delta f_1$ | $OS$ $\Delta f_2$ | $OS$ *tie* | $J_d$ ($\times 10^{-6}$) |
|---|---|---|---|---|---|---|---|---|---|---|
| **MET** | | | | | | | | | | |
| **Scenario 1** | 0.72906 | 0.15103 | 1.47965 | 10.885 | 11.528 | 8.620 | 0.2 | 0.26939 | 0.12867 | 4.7150 |
| **Scenario 2** | 0.72866 | 0.15095 | 1.47749 | 10.881 | 11.523 | 8.617 | 0.2 | 0.26943 | 0.12866 | 4.7120 |
| **Scenario 3** | 0.72954 | 0.15112 | 1.48154 | 10.887 | 11.530 | 8.621 | 0.2 | 0.26935 | 0.12867 | 4.7197 |
| **Scenario 4** | 0.72898 | 0.15103 | 1.47927 | 10.885 | 11.527 | 8.620 | 0.2 | 0.26937 | 0.12867 | 4.7151 |
| **Scenario 5** | 0.72925 | 0.15108 | 1.48066 | 10.887 | 11.530 | 8.621 | 0.2 | 0.26933 | 0.12867 | 4.7172 |
| **Scenario 6** | 0.72870 | 0.15098 | 1.47792 | 10.884 | 11.525 | 8.619 | 0.2 | 0.26941 | 0.12866 | 4.7125 |
| **DETC** | 0.73058 | 0.15146 | 1.48496 | 10.904 | 11.547 | 8.637 | 0.2 | 0.26914 | 0.12868 | 4.8570 |
| **SETC** | 0.73107 | 0.15158 | 1.48766 | 10.917 | 11.558 | 8.646 | 0.2 | 0.26913 | 0.12868 | 4.8326 |

The evolution of the states for Scenario 5 is depicted in Fig. 4, while Fig. 5 shows the MET control policies of players in the presence of continuous-time maximizing adversarial disturbance. The simulation runtime is set to 12 s with a time step of 0.001~s to ensure accurate numerical integration of the continuous-time dynamics and reliable evaluation of the triggering conditions.

Next, the proposed MET control approach is compared with the centralized dynamic event-triggered control (DETC) method in [19], where an event-triggered maximizing adversarial is considered. The method is selected because its multiplayer formulation accommodates multiple non-competitive and competitive players, making it structurally compatible with the considered MZS game. For completeness, a static event-triggered control (SETC) variant is also considered by setting the internal dynamic triggering variables to zero. The numbers of required policy updates of DETC and SETC schemes are reported in Table III. In addition, Table IV compares their closed-loop performance in terms of the integral absolute errors, the integral time absolute error, the settling times, the overshoots, and the disturbance-energy index ($J_d$). As shown in Table IV, the proposed MET approach achieves almost similar closed-loop performance to the DETC and SETC methods while considering a continuous-time adversarial player. As reported in Table III, the proposed MET control approach requires 740–774 policy updates across the considered scenarios, compared with 884 updates for DETC and 2692 updates for SETC. Therefore, for the considered application, the proposed approach reduces the number of updates by approximately 12–16% and 71–72% compared with DETC and SETC, respectively.

Unlike the approaches with centralized triggering mechanisms, the proposed approach allows decentralized triggering mechanism. The asynchronous triggering characteristics of the proposed MET strategy are further summarized in Table V, where the distribution of simultaneous triggering instants is reported for different MET scenarios. As reported, simultaneous triggering of all players occurs only rarely, while the majority of triggering instants involve a single player. In the rare case where all players trigger simultaneously, the proposed approach temporarily coincides with the behavior of a centralized triggering mechanism. Therefore, compared with centralized event-triggered approaches, the proposed decentralized triggering mechanism provides improved scalability in the sense that the instantaneous communication and policy update loads are determined by the subset of players satisfying their local triggering conditions rather than by the total number of players.

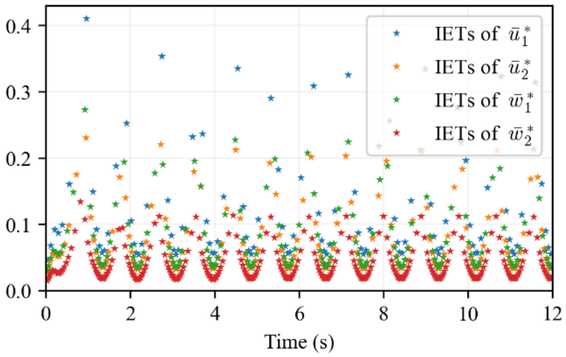



**Fig. 6.** Inter-event times of MET scenario 5.

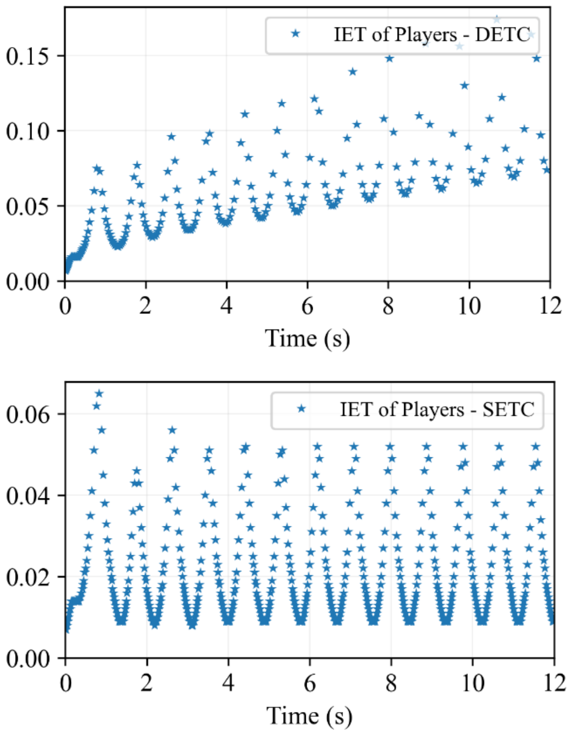



**Fig. 7.** Inter-event times for DETC and SETC.

The asynchronous triggering behaviour is also illustrated in Fig. 6, by the IETs of the proposed approach. The proposed MET control achieves longer IETs compared with the considered centralized methods (Fig. 7). Across the simulated scenarios, the maximum IET ranges from 0.33 to 0.41, showing an increase of at least 94% and 408% compared with the maximum IETs of 0.17 and 0.065 achieved by the DETC and SETC methods, respectively.

**Table V.** Asynchronous characteristics (MET)

| Scenario | Single instants | Double instants | Triple instants | Fully simultaneous |
|---|---|---|---|---|
| **1** | 712 (97.67%) | 16 (2.19%) | 0 (0.00%) | 1 (0.14%) |
| **2** | 704 (97.64%) | 15 (2.08%) | 1 (0.14%) | 1 (0.14%) |
| **3** | 728 (97.07%) | 21 (2.80%) | 0 (0.00%) | 1 (0.13%) |
| **4** | 694 (96.93%) | 21 (2.93%) | 0 (0.00%) | 1 (0.14%) |
| **5** | 709 (96.99%) | 20 (2.74%) | 1 (0.14%) | 1 (0.14%) |
| **6** | 695 (96.39%) | 25 (3.47%) | 0 (0.00%) | 1 (0.14%) |

## 7. Conclusion

This paper developed a decentralized MET control for linear MZS differential games. By enabling local triggering conditions, the proposed approach supports asynchronous control updates while attenuating adversarial disturbances and reducing unnecessary communication. Closed-loop stability and Zeno exclusion were established, and a systematic budget-allocation mechanism was proposed to design local triggering thresholds. Conceptual and numerical comparisons with centralized triggering methods demonstrated the effectiveness of the proposed approach.

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